

# Learning Goals and Accumulation Grading in Calculus I

## LEARNING GOALS

Here is a sample of learning goals from my calculus I class in Fall 2014.

### D-level goals

Learning Goal	A (1)	Q (2)	Q (3)	Q (4)
D6: I know when to use a limit.				
D7: I know when to use a derivative.				
D8: I know when to use an integral.				

### CD-level goals

Learning Goal	A (1)	Q (2)	Q (3)	Q (4)
CD3: I can estimate limits from graphs.				
CD5: I can estimate derivatives numerically.				
CD7: I can use Riemann sums to estimate integrals.				

### C-level goals

Learning Goal	A (1)	Q (2)	Q (3)	Q (4)
C1: I can identify the relationship between a function's value and its limit.				
C5: I can find derivatives using the definition of derivative.				
C6: I can sketch $f$ , $f'$ , or $f''$ given a graph of $f$ , $f'$ , or $f''$ .				

### BC-level goals

Learning Goal	A (1)	Q (2)	Q (3)	Q (4)
BC5: I can use the Product Rule.				
BC8: I can take derivatives of polynomials and rational functions.				
BC9: I can take derivatives of trigonometric functions.				
BC16: I can find antiderivatives of exponential functions.				
BC17: I can find the equation of a tangent line.				

### B-level goals

Learning Goal	A (1)	Q (2)	Q (3)	Q (4)
B2: I can use linear approximation.				
B3: I can find extrema.				
B6: I can determine intervals of concavity.				
B15: I can use FToCI.				
B16: I can use FToCII.				

### AB-level goals

Learning Goal	A (1)	Q (2)	Q (3)	Q (4)
AB1: I can write $\delta$ - $\epsilon$ proofs for linear functions.				

### A-level goals

Learning Goal	A (1)	Q (2)	Q (3)	Q (4)
A1: I can solve Related Rates problems.				
A2: I can write $\delta$ - $\epsilon$ proofs for quadratic functions.				

## ACCUMULATION GRADING

### Issues with Traditional Grading

- Grades lack intrinsic meaning.
- Students are expected to learn the course content at the same pace. Consider:
  - Student 1 gets 85% on all four exams.
  - Student 2 gets 100% on the first exam, 90% on the second, 80% on the third, and 70% on the fourth.
  - Student 3 gets 40% on the first exam, 100% on the second, 100% on the third, and 100% on the fourth.
- Grading does not give students feedback on how to improve.
- Grading is too "high stakes."
- Students encouraged to "bluff" for partial credit.
- Cramming is rewarded.
- There is not enough accountability to learning.

### Accumulation Grading

- All grading is binary: students either get 100% on the assignment, or they get 0% on the assignment.
- Students are given multiple chances to demonstrate mastery of Learning Goals
- Students must "tag" an answer with a Learning Goal in order to get credit for it (metacognitive benefits?).
- Students must demonstrate mastery over time

### Example

6. Find the equation of the line that is tangent to  $f(x) = e^x + x$  at  $(0, 1)$ .

$f'(x) = e^x + 1$   
 $f'(0) = e^0 + 1 = 1 + 1 = 2$   
 $y = 2x + b$   
 $1 = 2(0) + b$   
 $b = 1$   
 $y = 2x + 1$

### Benefits of Accumulation Grading

- Grades are based on what students know, rather than what they do not know.
- Students are allowed many chances to demonstrate mastery of material.
- Accountability: there is no substitute for learning the most important topics.
- Grades based on how much they know at *the end* of the semester

## ASSESSMENT

### Results

- Calculus Concept Inventory (CCI) pre- and post-test
- Similar to Force Concept Inventory (FCI) in physics
- Calculate "average normalized gains" of each class (the percentage of material the student learning *out of the material they did not already know*)

$$\langle g \rangle = \frac{\text{Final} - \text{Initial}}{\text{Total} - \text{Initial}}$$

- Sample question: If you know that a function  $f(x)$  is positive everywhere, what can you conclude from that about the derivative  $f'(x)$ ?
  - the derivative is positive everywhere
  - the derivative is increasing everywhere
  - the derivative is concave upward,
  - you can't conclude anything about the derivative

### Results

- Typical "Traditional Lecture" classes have average normalized gains of 0.08–0.23 on CCI
- Typical "Interactive Engagement" classes have average normalized gains of 0.30–0.44 on CCI
- Recent courses at CSBSJU ( $B$  indicates my class):

$n$	26	22	19	15	5	23	21	8	2	26	15	20	20
$\langle g \rangle$	0.29B	0.24B	0.14	0.15	0.19	0.19	0.15	0.09	0.03	0.01	0.09	0.21B	0.14B

### Bibliography

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- J. Epstein, The Calculus Concept Inventory—measurement of the effect of teaching methodology in mathematics, *Notices of the AMS*, **60**(8), 2013, 1018–1026.
- J. Epstein, "Calculus Concept Inventory," Field-tested Learning Assessment Guide, [http://www.flaguide.org/tools/diagnostic/calculus\\_concept\\_inventory.php](http://www.flaguide.org/tools/diagnostic/calculus_concept_inventory.php).

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