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The Force Exerted on a Sphere in a Tightly Focused Laser Beam

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The force exerted on a sphere in a tightly focused laser beam

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Abstract

An investigation of the trapping force on a sphere in a tightly focused beam is carried out using a geometrical optics approximation. This calculation generalizes earlier work by allowing the spheres to be located off the beam axis. Additionally, a fifth-order, instead of zeroth-order, gaussian approximation is used, and beam polarization is explicitly taken into account. The results obtained suggest an interesting experiment, involving a driven damped harmonic oscillator system, which could be performed to test the validity of the theory. This experiment is currently underway.

1 Introduction

In 1970 Arthur Ashkin first demonstrated the ability to manipulate micron-sized particles with radiation pressure from laser sources\(^1\). A simple model can be used to understand the origin of radiation pressure qualitatively. Consider the change in momentum of a light ray from a plane wave as it is scattered by a transparent sphere. As seen in Figure 1, a typical ray undergoes a change in momentum with a negative z component, where the z direction is defined by the direction of the incident beam propagation. If we sum over all rays incident on the sphere surface, axial symmetry
Figure 1: Change in momentum of a sphere due to a typical ray from a plane wave implies that the net change in momentum of the light is in the negative z direction. Thus, to conserve momentum, the sphere must undergo a change in momentum in the positive z direction. This phenomenon, where the sphere is pushed in the direction of beam propagation, is an example of radiation pressure.

In the same article Ashkin reported the trapping of particles in a stable "optical trap" using only the force of radiation pressure. Two equal strength, horizontal, counter-propagating beams provided the force balance required for the optical trap. For over a decade and a half after the publication of Ashkin's article there were numerous other investigations conducted concerning the stability of optical traps using radiation pressure\textsuperscript{2-7}. All of these investigations used multiple beams, or one beam along with an external force such as gravity, to obtain a potential well. It wasn't until 1986, that Ashkin, Dziedzic, Bjorkholm, and Chu\textsuperscript{8} realized the possibility of a
Figure 2: Change in momentum of a sphere due to rays from a focused beam.

stable optical trap using only one beam, with no need of an external force. The key is to use a tightly focused beam to obtain large axial gradient forces to balance the forward radiation pressure. Ashkin et al. termed this technique a single-beam gradient force trap.

Again, a simple model can be used to understand the gradient force trap qualitatively. The tightly focused beam can be approximated as a cone of light emanating from a point source. As seen in Figure 2, under these conditions the change in the sphere's momentum from ray A has a negative $z$ component, while from ray B it has a positive $z$ component. Again, symmetry requires that the net change in momentum of the sphere has only a $z$ component. However, whether this change is in the positive or negative $z$ direction depends upon how tightly the beam is focused, and how close the sphere is to the focus.
It is my intention in this thesis to make these arguments more quantitative. Using a geometrical optics approximation, I will directly calculate the force on a sphere in a tightly focused laser beam. I will then investigate the conditions necessary for optical trapping. It should be noted that a similar calculation has been performed recently by Gussgard, Lindmo and Brevik. However, there are important differences in the respective derivations. First, Gussgard et al. restrict themselves to spheres located on the beam axis, whereas I consider spheres located at any arbitrary point relative to the beam focus. Second, they used a zeroth-order gaussian approximation of the beam, whereas I use a fifth-order approximation. Finally, they did not explicitly take into account the effects of polarization, whereas I do. To the best of my knowledge the detailed calculation of the trapping force conducted in this thesis is the first to be generalized to include spheres located off the beam axis.

Section 2 gives a fairly detailed derivation of the trapping force. Section 3 describes the origin of the gradient force trap in more detail, and displays graphically the results of computations for a sphere of radius $a = 5 \, \mu m$ and index of refraction $n = 1.58$ immersed in water, illuminated by an argon laser beam of wavelength $\lambda = 488 \, nm$. The results obtained suggest an interesting experiment which could be performed to test the validity of the theory. This experiment is outlined in section 4, and is the focus of a current study. Appendices A and B include copies of the programs used in the computations.

2 Theory

Consider a beam with a gaussian profile. The beam width, $w$, is defined to be the distance from the beam axis at which the electric field amplitude is a fraction $\frac{1}{e}$ of
the maximum amplitude. According to the theory of gaussian beams\textsuperscript{10}, for a beam propagating in the \( +z \) direction with a minimum beam waist, \( w_0 \), located at the origin, the beam width is given by the equation

\[
w(z) = w_0 (1 + \frac{z^2}{g^2})^{\frac{1}{2}}
\]  

(1)

where \( g \) is a useful parameter defined by:

\[
g \equiv \frac{\pi w_0^2}{\lambda}
\]  

(2)

The minimum beam waist occurs at the beam focus. Thus the coordinate system is defined such that the origin corresponds to the beam focus, and the \(+z\) direction corresponds to the beam axis. Let the beam polarization be in the \(+x\) direction (see Figure 3).

Now consider a sphere of radius \( a \) centered at an arbitrary point, \( O \), with coordinates \((x_o, y_o, z_o)\) such that \( z_o > a \). Given a point on the sphere surface, we wish to trace the ray incident at that point through all possible reflections and refractions so as to determine the change in momentum of the ray. The sphere must undergo a change in momentum that is equal to but opposite that of the ray. Thus the force on the sphere due to the ray is the negative of the rate of change of the ray’s momentum. The total force is found by integrating over the sphere surface.

### 2.1 Unit Vectors

Let me begin by introducing some formalism. Consider a point \( M \) on the sphere surface, with coordinates \((x_m, y_m, z_m)\). The ray incident at \( M \) appears to emanate from the point \( C \), the center of curvature for the ray, which is located on the \( z \) axis a distance \(|b|\) from the origin (see Figure 3). The radius of curvature, \( R_m \), is given
Figure 3: Geometry for the calculation of the force on a sphere.

by the theory of gaussian laser beams\(^1\) as

\[ R_m = z_m + \frac{g^2}{z_m} \]  \( (3) \)

Geometrically, the radius of curvature of the beam at \( M \) is given by

\[ R_m = \sqrt{x_m^2 + y_m^2 + (z_m - b)^2} \]  \( (4) \)

The point \( C \) is then found by solving (4) for \( b \),

\[ b = z_m - \sqrt{R_m^2 - x_m^2 - y_m^2} \]  \( (5) \)

The points \( C, O, \) and \( M \) define the scattering plane for the ray incident to the sphere at \( M \).

Let \( \hat{u} \) be the unit vector in the direction of the the incident ray; \( \hat{n} \) the unit vector normal to the scattering plane; and \( \hat{p} \) the unit vector mutually perpendicular to \( \hat{u} \)
and \( \hat{n} \). In terms of the points \( C, O, \) and \( M \)

\[
\hat{u} \equiv \frac{\overrightarrow{CM}}{|\overrightarrow{CM}|} \\
\hat{n} \equiv \frac{\overrightarrow{CO} \times \overrightarrow{CM}}{|\overrightarrow{CO} \times \overrightarrow{CM}|} \\
\hat{p} \equiv \hat{n} \times \hat{u}
\]

(6)

2.2 Electric field

If polarization is to be taken into account explicitly, the electric field vector at the point \( M \) is needed. The often-used zeroth-order gaussian approximation, where the electric field at a point \((x, y, z)\) is given by

\[
E = \frac{2}{w} \sqrt{\frac{P}{w^3 \pi}} \exp \left[ -\frac{(x^2 + y^2)}{w^2} \right]
\]

(7)

where \( P \) is the beam power, \( v \) is the speed of light in the medium, and \( \epsilon \) is the permittivity of the medium, becomes increasingly inaccurate for smaller beam waists. We will find that beam waists on the order of the wavelength are necessary for optical trapping to occur. Thus higher-order terms must be included to describe the electric field accurately.

Barton and Alexander\textsuperscript{11} recently derived the correction terms to the gaussian approximation up to fifth order in the dimensionless parameter

\[
s \equiv \frac{\lambda}{2\pi w_o}.
\]

(8)

To this order, the components of the electric field vector at the point \( M \) on the sphere surface are given by
\[ E_x = \{1 + (sQ)^2(-\rho^2 + i\rho^4Q - 2x_n^2) + (sQ)^4[2\rho^4 - 3i\rho^6Q - \rho^8Q^2/2 + x_n^2(8\rho^2 - 2i\rho^4Q)]\}E_t \]
\[ E_y = x_ny_n[-2(sQ)^2 + (sQ)^4(8\rho^2 - 2i\rho^4Q)]E_t \]
\[ E_z = x_n[-2(sQ) + (sQ)^3(6\rho^2 - 2i\rho^4Q) + (sQ)^5(-20\rho^4 + 10i\rho^6Q + \rho^8Q^2)]E_t \]  
(9)

where \( s \) is defined as above, and

\[ E_t \equiv \frac{2}{w} \left[ \frac{P}{v\varepsilon\pi(1 + 2s^2 + \frac{11}{2}s^4)} \right]^{\frac{1}{2}} \exp \left[ \frac{-(x_m^2 + y_m^2)}{w^2} \right], \]  
(10)

\[ \begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix} \equiv \frac{1}{w_o} \begin{pmatrix} x_m \\ y_m \\ s z_m \end{pmatrix}, \]  
(11)

\[ \rho \equiv \sqrt{x_n^2 + y_n^2}, \]  
(12)

\[ Q \equiv g \frac{(x_m - ig)}{(x_m^2 + g^2)}. \]  
(13)

It should be noted that this approximation introduces a small uncertainty into my calculation. Barton and Alexander calculated the relative uncertainties for the zeroth- through fifth-order approximations for various values of \( s \). My calculations assume an argon beam of wavelength \( \lambda \approx 367 \text{ nm} \) in water, focused to a minimum beam waist of \( w_o \approx .2 \mu\text{m} \). Thus, \( s \approx .3 \) for my calculations. According to Barton and Alexander, for this value of \( s \) the average uncertainty for the fifth-order approximation is about 0.725\%: the results of my computations using this approximation will have a corresponding uncertainty. In comparison, the average uncertainty for the zeroth-order approximation is about 15.3\%.
2.3 Transmission Coefficients

When the light ray reaches the sphere it is scattered by multiple reflections and refractions. The fraction of the electric field amplitude reflected at each interface is given by the Fresnel equations\textsuperscript{12}

\[ r_n = -\frac{\sin(\alpha - \beta)}{\sin(\alpha + \beta)} \]

\[ r_p = -\frac{\tan(\alpha - \beta)}{\tan(\alpha + \beta)} \] (14)

where \( \alpha \) is the angle of incidence, and \( \beta \) is the angle of refraction. The subscripts \( n \) and \( p \) refer to the reflection coefficients for light polarized normal to and parallel to the scattering plane respectively.

The incidence angle is easily determined by the dot product of \( \hat{u} \) and \( \vec{OM} \):

\[ \alpha = \arccos \left( \frac{-\hat{u} \cdot \vec{OM}}{|\vec{OM}|} \right) \] (15)

The refraction angle is also easily determined by Snell’s Law\textsuperscript{12}:

\[ \beta = \arcsin \left( \frac{\sin \alpha}{m} \right) \] (16)

where \( m \) is the index of refraction of the sphere relative to the surrounding liquid. The perpendicular and parallel components of the electric field vector, \( E_n \) and \( E_p \), are given by the scalar product of \( \vec{E} \) with the unit vectors \( \hat{n} \) and \( \hat{p} \) respectively.

At each interface a fraction \( r_j^2 \) of the light energy is reflected (squared because intensity is proportional to the square of the electric field amplitude), and a fraction \( (1 - r_j^2) \) is transmitted (where \( j = n \) or \( p \)). Figure 4 follows a ray through a number of reflections and refractions, and lists the fraction of light energy retained for each
Figure 4: Diagram giving transmission coefficient and exit angle as a function of chord number, \( h \).

Ray. From this figure it is seen that the fraction of light energy transmitted through the sphere, \( k_j \), is given as a function of chord number, \( h \), by

\[
k_j = \begin{cases} 
  r_j^2 & \text{if } h = 0 \\
  r_j^{2(h-2)}(1 - r_j^2)^2 & \text{if } h \geq 1 
\end{cases}
\]  

(17)

2.4 Exit Angles

With the same figure (Figure 4), we can derive an expression relating the chord number and \( \delta \), the exit angle with respect to the unit vector \( \hat{u} \). The exit angle is found by subtracting from \( \pi \) the incidence angle \( \alpha \), the angle subtended in the sphere, and the final refraction angle (which is also \( \alpha \)). As seen in the figure, the
angle subtended in the sphere is $h(\pi - 2\beta)$. Thus we obtain the relation

$$\delta = \pi(1-h) - 2\alpha + 2h\beta.$$  \hspace{1cm} (18)

### 2.5 Force on sphere due to a single ray

We are now prepared to determine the change in momentum of the ray incident at the point $M$ on the sphere surface. The incident power on the surface patch at $M$ is simply

$$I_j (\cos \alpha) dA$$  \hspace{1cm} (19)

where $I_j$ is the light intensity given by

$$I_j = \frac{\nu \epsilon}{2} E_j E_j^*,$$  \hspace{1cm} (20)

where $E_j^*$ denotes the complex conjugate of $E_j$, and again $\nu$ is the speed of light in the medium, and $\epsilon$ the permittivity of the medium. The ray travels toward the sphere in the $\hat{\nu}$ direction. Thus the initial momentum of the ray for a time $\Delta t$ is

$$\vec{p}_i = \sum_{j=n,p} \frac{\Delta t}{\nu} I_j (\cos \alpha)(dA) \hat{\nu}.$$  \hspace{1cm} (21)

Similarly the scattered rays have momentum

$$\vec{p}_s = \sum_{j=n,p} \frac{\Delta t}{\nu} I_j k_j (\cos \alpha)(dA)(\hat{p} \sin \delta + \hat{\nu} \cos \delta)$$  \hspace{1cm} (22)

where both $k_j$ and $\delta$ are functions of $h$. If we make this $h$ dependence explicit, we have for the first reflected ray ($h = 0$)

$$\vec{p}_r = \sum_{j=n,p} \frac{\Delta t}{\nu} I_j r_j^2 (\cos \alpha)(dA)(\hat{p} \sin 2\alpha - \hat{\nu} \cos 2\alpha)$$  \hspace{1cm} (23)
and for the transmitted rays \((h \geq 1)\)

\[
\tilde{p}_t = \sum_{j=n,p} \frac{\Delta t}{\nu} I_j(\cos \alpha)(dA) (1 - r_j^2) r_j^{-2} \left[ \hat{u} \sum_{h=1}^{\infty} r_j^{2h} \cos \delta \\
+ \hat{p} \sum_{h=1}^{\infty} r_j^{2h} \sin \delta \right]. \tag{24}
\]

The infinite summations in (24) are easily performed if we use the complex expressions for the trigonometric functions:

\[
\sum_{h=1}^{\infty} r_j^{2h} \cos \delta = \sum_{h=1}^{\infty} r_j^{2h} \cos(\pi(1 - h) - 2\alpha + 2h\beta)
= \sum_{h=1}^{\infty} r_j^{2h} \left( \frac{e^{i(\pi - 2\alpha)} e^{i h (2\beta - \pi)} + e^{-i(\pi - 2\alpha)} e^{-i h (2\beta - \pi)}}{2} \right)
= \frac{e^{i(\pi - 2\alpha)}}{2} \sum_{h=1}^{\infty} r_j^{2h} e^{i h (2\beta - \pi)} + \frac{e^{-i(\pi - 2\alpha)}}{2} \sum_{h=1}^{\infty} r_j^{2h} e^{-i h (2\beta - \pi)}. \tag{25}
\]

Each of the summations in the last line of (25) is a geometric series with initial term and ratio \(r = r_j^2 \exp(\pm i (2\beta - \pi))\). Since \(r < 1\), these summations converge, and we can thus write them in closed form to obtain

\[
\sum_{h=1}^{\infty} r_j^{2h} \cos \delta = \frac{r_j^2}{2} \left[ \frac{e^{i(\pi - 2\alpha)} e^{i (2\beta - \pi)}}{1 - r_j^2 e^{i (2\beta - \pi)}} + \frac{e^{-i(\pi - 2\alpha)} e^{-i (2\beta - \pi)}}{1 - r_j^2 e^{-i (2\beta - \pi)}} \right]
= \frac{r_j^2}{2} \left[ \frac{e^{i(\pi - 2\alpha)} e^{i (2\beta - \pi)} + e^{-i(\pi - 2\alpha)} e^{-i (2\beta - \pi)}}{1 + r_j^2 (e^{i (2\beta - \pi)} + e^{-i (2\beta - \pi)})} \right]
= r_j^2 \left[ \frac{\cos(2\beta - 2\alpha) + r_j^2 \cos 2\alpha}{1 + 2r_j^2 \cos 2\beta + r_j^4} \right] \equiv r_j^2 \xi_j. \tag{26}
\]

Likewise

\[
\sum_{h=1}^{\infty} r_j^{2h} \sin \delta = r_j^2 \left[ \frac{\sin(2\beta - 2\alpha) - r_j^2 \sin 2\alpha}{1 + 2r_j^2 \cos 2\beta + r_j^4} \right] \equiv r_j^2 \eta_j. \tag{27}
\]
Using the results (26) and (27), the total change in momentum of the light is

\[
\Delta \vec{p}_{ray} = (\vec{p}_r + \vec{p}_t) - \vec{p}_i \\
= \sum_{j=n,p} \frac{\Delta t}{v} I_j(\cos \alpha)(dA) \left\{ \hat{u} \left[ (1 - r_j^2)^2 \xi_j - r_j^2 \cos 2\alpha - 1 \right] \\
+ \hat{p} \left[ (1 - r_j^2)^2 \eta_j + r_j^2 \sin 2\alpha \right] \right\}.
\]

(28)

Thus the force on the sphere due to the ray is

\[
d\vec{F} = -\frac{\Delta \vec{p}_{ray}}{\Delta t} \\
= \sum_{j=n,p} \frac{1}{v} I_j(\cos \alpha)(dA) \left\{ \hat{u} \left[ 1 + r_j^2 \cos 2\alpha - (1 - r_j^2)^2 \xi_j \right] \\
+ \hat{p} \left[ -r_j^2 \sin 2\alpha - (1 - r_j^2)^2 \eta_j \right] \right\} \\
= \frac{1}{v} (\cos \alpha)(dA) \left[ \hat{u}(I_n u_n + I_p u_p) + \hat{p}(I_n p_n + I_p p_p) \right].
\]

(29)

where

\[
u_j \equiv \left[ 1 + r_j^2 \cos 2\alpha - (1 - r_j^2)^2 \xi_j \right],
\]

\[
p_j \equiv \left[ -r_j^2 \sin 2\alpha - (1 - r_j^2)^2 \eta_j \right].
\]

(30)

The total force on the sphere is then obtained by integrating \(d\vec{F}\) over the sphere surface.

### 2.6 Force of beam on sphere

It is convenient to introduce a new coordinate system to define the limits of integration. The obvious choice is a spherical coordinate system centered at the sphere’s center \(O\), with the \(z\) axis passing through the beam focus, i.e. the origin of the coordinate system used heretofore (see Figure 5). With this system, the limits of
integration then become the normal limits; i.e. $0 \leq \theta \leq \frac{\pi}{2}$, and $0 \leq \phi \leq 2\pi$, with $dA = a^2 \sin \theta \, d\theta \, d\phi$. The total force on the sphere is then given by

$$\vec{F} = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \frac{a^2}{v} \cos \alpha \sin \theta \, d\theta \, d\phi \left[ \dot{u}(I_n u_n + I_{\rho u_p}) + \dot{\rho}(I_n p_n + I_{\rho p_p}) \right]. \quad (31)$$

To obtain the transformation from sphere-centered coordinates to beam-centered coordinates, a series of rotations and translations were performed: (i) rotation of the beam coordinates about the $z$ axis by the angle $\arctan(\frac{y_0}{z_0})$, (ii) a second rotation about the new $y$ axis by the angle $\arctan(\frac{z_0}{r_0})$, where $r_0 \equiv \sqrt{x_0^2 + y_0^2}$ and $d_0 \equiv \sqrt{r_0^2 + z_0^2}$, (iii) a third rotation about the new $x$ axis by $\pi$, and (iv) a translation in the $-z$ direction of $d_0$ units. The inverse of the product of these rotation matrices gives us the desired transformation.
\[ x_m = \frac{x_0 z_0 x}{r_0 d_o} + \frac{y_0 y}{r_o} - \frac{z_0 z}{d_o} + x_o \\
y_m = \frac{y_0 z_0 x}{r_0 d_o} + \frac{x_0 y}{r_o} - \frac{y_0 y}{d_o} + y_o \\
z_m = \frac{r_0 x}{d_o} - \frac{z_0 z}{d_o} + z_o \]  

(32)

where \( x, y, \) and \( z \) are given by the spherical polar coordinates in the usual way

\[ x = a \sin \theta \cos \phi \]
\[ y = a \sin \theta \sin \phi \]
\[ z = a \cos \theta . \]  

(33)

3 Results

The force element \( d\vec{F} \) (Equation 29) was written to a FORTRAN external program file (see Appendix A). With this program numerical calculations can be performed to show more quantitatively how trapping occurs, thus complementing the discussion given in the Introduction. All computations were performed assuming a sphere of radius 5 \( \mu m \) immersed in water, illuminated by a 1 Watt argon laser beam of wavelength 488nm. Figure 6 shows a plot of \( dF_z \) versus \( \theta \), holding \( \phi \) fixed at 0, for a sphere centered on the \( z \) axis for various values of \( z_o \), with beam waist \( w_o = .2 \mu m \). This plot displays the force contribution of a strip down the front of the sphere. Positive values denote forward radiation pressure, while negative values denote the gradient force.

From the figure we see that we obtain the strongest contributions to the gradient force for small values of \( z_o \). As the sphere moves away from the beam focus, a larger portion of the sphere becomes illuminated, and the outside region begins to dominate. However, in the trapping region, i.e., the region which displays a net
Figure 6: The force contribution for a strip down the front face of a sphere at distances of (a) \( z_0 = 6.5 \mu m \), (b) \( z_0 = 8 \mu m \), (c) \( z_0 = 10 \mu m \), and (d) \( z_0 = 12 \mu m \). These calculations were performed for a beam waist of \( w_0 = 0.2 \mu m \).

Negative force contribution, the rays near the beam center contribute the most to the forward radiation pressure. This phenomenon becomes more evident for larger beam waists. Figure 7 shows similar plots for beam waists of \( w_0 = 0.3 \mu m \) and \( w_0 = 0.4 \mu m \). These figures suggest that the trapping force can be greatly enhanced by inserting a conical field stop into the light path. Gussgard et al. calculated such an effect in their study, and found noticeable improvement in the strength of the trapping force, especially for larger beam waists.

Using a second FORTRAN program (see Appendix B) to numerically integrate the force element \( d\vec{F} \), I obtained plots of the trapping force, \( F_z \), versus distance from the beam focus, \( z_0 \). Figure 8 shows \( F_z \) versus \( z_0 \) for a sphere located on the beam axis for four beam waists between .2 \( \mu m \) and .4 \( \mu m \). Here positive values denote a force
Figure 7: The force contribution for a strip down the front face of a sphere for (a) $w_o = .3 \mu m$, $z_o = 8 \mu m$, and (b) $w_o = .4 \mu m$, $z_o = 10 \mu m$.

in the direction of beam propagation, while negative values denote a pull toward the bean focus. A stable trap is possible if the curve crosses zero with a negative slope: a displacement in the positive $z$ direction results in a restoring force in the negative $z$ direction, while a displacement in the negative $z$ direction results in a restoring force in the positive $z$ direction. We see from the figure that a beam waist of less than $w_o \cong .33 \mu m$ is necessary for optical trapping (although there is a theoretical minimum beam waist of $w_o \cong \frac{\lambda}{2}^{13}$, which corresponds to $w_o \cong .18 \mu m$ for an argon beam of wavelength $\lambda = .488 \mu m$ in water).

It should be noted that these results are not in agreement with those obtained by Gussgard et al. In their calculations they found $F_z$ to be positive for all values of $z_o$ for beam waists greater than $w_o \cong .3 \mu m$. I believe the discrepancy is caused by their
Figure 8: Plots of $F_x$ versus $z_o$ for beam waists of (a) $w_o = .4 \mu m$, (b) $w_o = .33 \mu m$, (c) $w_o = .3 \mu m$, and (d) $w_o = .2 \mu m$. These calculations were performed using the fifth-order gaussian approximation.

use of a zeroth-order approximation for a gaussian beam. My results of $F_x$ versus $z_o$ using a zeroth-order approximation are consistent with those given by Gussgard et al. Figure 9 combines my results for the zeroth-order and fifth-order approximations for three values of the beam waist. The figure confirms the earlier claim that the zeroth-order approximation becomes increasingly inaccurate for smaller beam waists, which are the beam waists of interest for optical trapping. Thus I feel that it is necessary to include the higher-order approximation to describe the trapping force accurately.

One other difference in my derivation compared with that of Gussgard et al. is the inclusion of the effects of beam polarization. I suspected that polarization effects would produce noticeable differences in the forces $F_x$ and $F_y$ needed to push a sphere out of a trap in the $x$ and $y$ directions respectively. Figure 10 displays $F_x$ versus $x_o$, 

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Figure 9: Plots of $F_z$ versus $z_o$ for beam waists of (a) $w_o = .4 \, \mu m$, (b) $w_o = .3 \, \mu m$, and (c) $w_o = .2 \, \mu m$. The primed plots correspond to the calculations performed using the zeroth-order gaussian approximation, while the unprimed plots correspond to those performed using the fifth-order approximation.

and $F_y$ versus $y_o$ for a beam waist of $w_o = .2 \, \mu m$. Here $z_o = 5.4 \, \mu m$, which roughly corresponds to the location of a stable optical trap for a beam propagating vertically downward, where the gradient force is balanced by gravity. As seen in the figure, there are no noticeable effects for displacements less than about 2.5 $\mu m$. For larger displacements the polarization effects are noticeable, although they are quite small.

4 Possible Experiment

The results displayed in Figure 10 suggest an interesting experiment which could be performed to test the validity of the theory. For displacements less then about 3 $\mu m$ off the beam axis, the restoring force is linear. Thus, according to this theory, a trapped
Figure 10: Plots of (a) $F_y$ versus $y_o$, and (b) $F_z$ versus $x_o$ for $x_o = 5.4 \, \mu m$, and $w_o = 0.2 \mu m$. Both calculations were performed using the fifth-order gaussian approximation.

The sphere should undergo oscillatory motion if it is displaced in a direction perpendicular to the beam axis. Since the sphere is immersed in a liquid, which provides a resistive force, this motion is described by the solution to the equation of motion

$$m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = 0$$

(34)

where $m$ is the mass of the sphere, $b$ is a constant giving the strength of the damping, and $k$ is the spring constant of the system. From fluid dynamics\textsuperscript{14}, for a sphere traveling through a viscous medium

$$b = 6\pi \eta r$$

(35)

where $\eta$ is the viscosity of the liquid, and $r$ is the radius of the sphere.
The solution to (34) is given by\(^\text{15}\)

\[ x(t) = A \exp\left(\frac{-\gamma t}{2}\right) \cos(\omega t + \alpha) \]  

where

\[ \gamma \equiv \frac{b}{m} \]

and

\[ \omega^2 = \omega_0^2 - \frac{\gamma^2}{4} \equiv \frac{k}{m} - \frac{b^2}{4m^2} \]  

provided that

\[ \omega_0^2 > \frac{\gamma^2}{4}. \]  

The oscillatory motion described by (36) can be used to measure the spring constant \(k\) experimentally. One possible experimental arrangement is shown in Figure 11. A tightly focused beam propagating vertically downward is used to trap a sphere in stable equilibrium. A lens is then arranged such that the sphere is located at the focal point of the lens. Radiation pressure from a second beam, with a relatively large beam waist, is used to displace the sphere horizontally. With the removal of the displacement beam the sphere will begin to oscillate in accordance with (36). Since the amplitude decays with time, it will be periodically necessary to re-displace the sphere. This re-displacement can be accomplished by using a spinning mirror, which revolves with frequency \(\omega' < \omega\), to direct the displacement beam. As the sphere oscillates through its equilibrium position, a flash of light from the scattered trapping beam passes through the lens to an electronic sensor which is connected to an oscilloscope. The frequency \(\omega\) can be obtained from the oscilloscope trace. The value of \(k\), i.e. the slope of the restoring force versus displacement, can then be determined from (37).
Figure 11: The apparatus for a possible experiment to measure the slope of the restoring force versus displacement.

It should be noted that for the calculations performed in section 3, the inequality (38) is not satisfied: the calculations assumed a latex sphere of radius $a = 5 \mu m$ and specific gravity $\sim .9$, immersed in water. These values yield $\omega_0 \cong 25 \text{ kHz}$ and $\gamma \cong 200 \text{ kHz}$. However, section 2 shows that the force on the sphere is proportional to the beam power; i.e., the intensity is proportional to the square of the electric field which is proportional to the square root of the beam power. Thus $k$, and hence $\omega_0$, can be increased by increasing the beam power. From (35) we see that $\gamma$ is proportional to the viscosity of the liquid. Thus $\gamma$ can be reduced by using a less viscous medium. Hence under suitable conditions the above inequality may be satisfied. Unfortunately, at the present time I do not have the equipment necessary to perform such an experiment.
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Appendix

A Program pt

PROGRAM pt
C
*******************************************************************************
* Program designed to estimate the integral of dfz by means of the Monte Carlo method
*******************************************************************************
C
REAL*8 zo, Pi, THETA, FZ, del, dfz
INTEGER M, N, J, 1
External dfz
C
Pi = 4.0D0*DATAN(1.0D0)
C
OPEN (30,FILE='p6.dat',STATUS='NEW',IOSTAT=M)
IF (M.NE.0) THEN
   WRITE (*,'(/1X,A)') 'ERROR OPENING FILE'
ENDIF
C
Calculates the function value for 200 points between -Pi/2 and Pi/2
C
N = 200
theta = -Pi/2
del = Pi/200
DO J=1,N
   fz=dfz(THETA)
   IF (NOT.(fz.ge.0)).AND.(NOT.(fz.lt.0)) GO TO 50
C
C Writes these values to a file in pairs (theta, dfZ)
C
   WRITE (30,*) theta, FZ
   theta = theta + del
C
ENDDO
C
100 CLOSE (30,STATUS='KEEP',IOSTAT=M)
IF (M.NE.0) THEN
   WRITE (*,'(/1X,A)') 'ERROR CLOSING FILE'
ENDIF
END
FUNCTION dfz(THETA)

* COMPUTES FORCE OF RAY INCIDENT ON SPHERE AT THETA, PHI *

REAL*8 Pi, Deg, a, ns, nl, m, lambdao, lambda, wo, eo, g, s,
& phi, theta, xo, yo, zo, ro, do, x, y, z, xm, ym, zm,
& rm, b, oml, om2, om3, cm1, cm2, cm3, u1, u2, u3, ang,
& alfa, beta, co1, co2, co3, v1, v2, v3, perl, per2,
& per3, n1, n2, n3, par1, par2, par3, pl, p2, p3, xn,
& yn, zn, rho, w, in, ip, rn, rp, dp, etal, eta2,
& eta3, eta4, un, up, pn, pp, dfz, et

COMPLEX*16 I, q, ex1, ex2, ex3, ey, ez1, ez2, ez, en, ep

PI = 4.0D0*DATAN(1.0D0)
Deg = PI/180.0D0
I = (0.0D0, 1.0D0)

C Initializes the variables such as sphere radius, refractive index of the sphere and liquid, the wavelength, etc.
C
a = 5.0D0
ns = 1.58D0
nl = 1.33D0
m = ns/nl
lambdao = 0.488D0
lambda = lambdao/nl
wo = .33D0
eo = 1.0D0
g = PI*wo**2/lambda
s = lambda/(2.0D0*PI*wo)

C Sets phi equal to zero
C
phi = .000001
C
C Defines the relative position to the beam axis
C
yo = 1.0D-6
xo = 0.0D0
zo = 10D0
ro = DSQRT(xo**2 + yo**2)
do = DSQRT(ro**2 + zo**2)
C
C Performs the transformation from theta and phi in sphere coordinates to (x,y,z) in beam coordinates
C
x = a*DSin(theta)*DCos(phi)
y = a*DSin(theta)*DSin(phi)
z = a*DCos(theta)
\[\begin{align*}
xm &= xo^2*yo^2/(ro*do) + yo^2/ro - xo^2/ro + xo \\
yn &= yz^2/ro^2/(ro*do) - xo^2*yo/ro - yo^2/ro + yo \\
zm &= \frac{-ro^2*yo}{zo^2} - zo^2/zo + zo \\
rm &= zm + g^2/zo
\end{align*}\]

C
Finds the point C

\[b = zm - DSQRT(rm^2 - zm^2 - ym^2)\]

C
Defines the vectors CM and CM and unit vector u

\[\begin{align*}
cm1 &= xm-xo \\
cm2 &= ym-yo \\
cm3 &= zm-zo \\
cm1 &= zm \\
cm2 &= ym \\
cm3 &= zm-b \\
u1 &= cm1/DSQRT(cm1*cm1 + cm2*cm2 + cm3*cm3) \\
u2 &= cm2/DSQRT(cm1*cm1 + cm2*cm2 + cm3*cm3) \\
u3 &= cm3/DSQRT(cm1*cm1 + cm2*cm2 + cm3*cm3)
\end{align*}\]

C
Determines if incidence angle is physically possible

\[\begin{align*}
IF &\ (-cm1*u1 + cm2*u2 + cm3*u3)/a .GT. 1.D0) THEN \\
dfz &= 0.D0 \\
GOTO 500 \\
ENDIF
\end{align*}\]

C
Determines if the incidence angle is less than Pi/2

If so program continues

If not program returns the value dfz=0

\[\begin{align*}
ang &= DAcos\left(-cm1*u1 + cm2*u2 + cm3*u3\right)/a \\
IF &\ (ang .LE. Pi/2.0) THEN \\
alfa &= ang
\end{align*}\]

C
Calculates the refraction angle beta

\[\beta = DAcos(DSin(alfa)/m)\]

C
Defines the vectors CO, v, n, and p

\[\begin{align*}
col &= xo \\
co2 &= yo \\
co3 &= zo-b \\
v1 &= col/DSQRT(col*col + co2*co2 + co3*co3) \\
v2 &= co2/DSQRT(col*col + co2*co2 + co3*co3) \\
v3 &= co3/DSQRT(col*col + co2*co2 + co3*co3) \\
per1 &= b*(ym-yo) - ym*zo + yo*zm \\
per2 &= b*(xo-xm) + xm*zo - xo*zm \\
per3 &= xo*ym-xm*yo \\
n1 &= per1/DSQRT(per1*per1 + per2*per2 + per3*per3) \\
n2 &= per2/DSQRT(per1*per1 + per2*per2 + per3*per3) \\
n3 &= per3/DSQRT(per1*per1 + per2*per2 + per3*per3)
\end{align*}\]
\[ \begin{align*}
\text{par1} &= b^{*2}(x_m-x_o) + b^{*}(2.0x_0z_m-x_m^{*}(z_m+z_0)) + \\
& \\
\text{par2} &= b^{*2}_y(y_m-y_0) + b^{*}(2.0y_0z_m-y_m^{*}(z_m+z_0)) - \\
& \\
\text{par3} &= b^{*2}(x_m^2-x_m+y_m^{*}(y_m-y_0)) - x_m^{*2}z_o + x_m^{*}x_oz_m + \\
& \\
\text{pl} &= \text{par1}/\text{DSqr}(\text{par1}*\text{par1} + \text{par2}*\text{par2} + \text{par3}*\text{par3}) \\
\text{p2} &= \text{par2}/\text{DSqr}(\text{par1}*\text{par1} + \text{par2}*\text{par2} + \text{par3}*\text{par3}) \\
\text{p3} &= \text{par3}/\text{DSqr}(\text{par1}*\text{par1} + \text{par2}*\text{par2} + \text{par3}*\text{par3}) \\
\text{C} & \quad \text{Calculates the electric field vector components} \\
\text{xn} &= x_m/w_o \\
\text{yn} &= y_m/w_o \\
\text{zn} &= s^{*}z_m/w_o \\
\text{rho} &= \text{DSqr}(x_n^{*2} + y_n^{*2}) \\
\text{w} &= w_0^{*}\text{DSqr}(1.0 + (z_m/g)^{**2}) \\
\text{q} &= g^{*}(z_m - I*g)/(z_m^{*2} + g^{**2}) \\
\text{et} &= (e_0^{*}w_0^{*}\text{DExp}(-(x_m^{*2} + y_m^{*2})/w^{**2})/w \\
\text{ex1} &= ((s^{*}q)^{**2})*(\text{rho}^{*2} + I*q*rho^{**4} - 2.0*x_n^{*2}) \\
\text{ex2} &= 8.0*rho^{*2} - 2.0*I*q*rho^{**4} \\
\text{ex3} &= ((s^{*}q)^{**4})*(2.0*rho^{*4} - 3.0*I*q*rho^{**6} - \\
& \\
\text{ex} &= (1 + ex_1 + ex_2)*et \\
\text{ey} &= x_n*y_n^{*}(-2.0*(s^{*}q)^{**2} + ((s^{*}q)^{**4})*(8.0*rho^{**2} - \\
& \\
\text{ez1} &= ((s^{*}q)^{**3})*(6.0*rho^{*2} - 2.0*I*q*rho^{**4}) \\
\text{ez2} &= ((s^{*}q)^{**5})*(-20.0*rho^{*4} + 10.0*I*q*rho^{**6} + \\
& \\
\text{ez} &= x_n^{*}(-2.0*(s^{*}q) + ez_1 + ez_2)*et \\
\text{C} & \quad \text{Calculates the components perpendicular and parallel to} \\
\text{C} & \quad \text{the scattering plane} \\
\text{en} &= ex^{*}n_1 + ey^{*}n_2 + ez^{*}n_3 \\
\text{ep} &= ex^{*}p_1 + ey^{*}p_2 + ez^{*}p_3 \\
\text{C} & \quad \text{Calculates the Intensities, less the proportionality constant} \\
\text{in} &= \text{DREAL}(en^{*}\text{DCONJG}(en)) \\
\text{ip} &= \text{DREAL}(ep^{*}\text{DCONJG}(ep)) \\
\text{C} & \quad \text{Defines the reflection coefficients} \\
\text{rn} &= \text{DSIN}(\text{alfa} - \beta)/\text{DSIN}(\text{alfa} + \beta) \\
\text{rp} &= \text{DTAN}(\text{alfa} - \beta)/\text{DTAN}(\text{alfa} + \beta) \\
\text{C} & \quad \text{Defines eta and xi} \\
\text{iv}


\[
\begin{align*}
\text{dn} &= 1.0 + (2.0 * \text{rn}**2) * \text{DCOS}(2.0 * \text{beta}) + \text{rn}**4 \\
\text{dp} &= 1.0 + (2.0 * \text{rp}**2) * \text{DCOS}(2.0 * \text{beta}) + \text{rp}**4 \\
\text{eta1} &= \left(\text{DCOS}(2.0 * \text{beta} - 2.0 * \text{alfa}) + (\text{rn}**2) * \text{DCOS}(2.0 * \text{alfa})\right) / \text{dn} \\
\text{eta2} &= \left(\text{DCOS}(2.0 * \text{beta} - 2.0 * \text{alfa}) + (\text{rp}**2) * \text{DCOS}(2.0 * \text{alfa})\right) / \text{dp} \\
\text{eta3} &= (\text{DSIN}(2.0 * \text{beta} - 2.0 * \text{alfa}) - (\text{rn}**2) * \text{DSIN}(2.0 * \text{alfa})) / \text{dn} \\
\text{eta4} &= (\text{DSIN}(2.0 * \text{beta} - 2.0 * \text{alfa}) - (\text{rp}**2) * \text{DSIN}(2.0 * \text{alfa})) / \text{dp}
\end{align*}
\]

C
C Defines \(u_n\), \(u_p\), \(p_n\), and \(p_p\)
C

\[
\begin{align*}
\text{un} &= 1.0 + (\text{rn}**2) * \text{DCOS}(2.0 * \text{alfa}) - ((1.0 - \text{rn}**2)**2) * \text{eta1} \\
\text{up} &= 1.0 + (\text{rp}**2) * \text{DCOS}(2.0 * \text{alfa}) - ((1.0 - \text{rp}**2)**2) * \text{eta2} \\
\text{pn} &= (-\text{rn}**2) * \text{DSIN}(2.0 * \text{alfa}) - ((1.0 - \text{rn}**2)**2) * \text{eta3} \\
\text{pp} &= (-\text{rp}**2) * \text{DSIN}(2.0 * \text{alfa}) - ((1.0 - \text{rp}**2)**2) * \text{eta4}
\end{align*}
\]

C
C Calculates the force element \(dF_z\) with the proportionality constant

\[
\text{dfz} = (20.2235 / (\text{wo}**2 * (1 + 2 * s**2 + 5.5 * s**4))) * (a**2) * \text{DCOS}(\text{alfa}) * ((\text{in} * \text{un} + \text{ip} * \text{up}) * \text{u3} + \text{in} * \text{pn} + \text{ip} * \text{pp}) * \text{DABS}(\text{DSIN}(\text{theta}))
\]

\[
\text{ELSE}
\text{dfz} = 0.0D0
\text{END IF}
\]

500 END

B Program intz

PROGRAM INTZ
C
**********************************************************************************************************************
*                                                                                                                     *
* Program designed to estimate the integral of \(dF_z\) by means of the Monte Carlo method                          *
**********************************************************************************************************************
C
REAL*8 zo, Pi, SUM, AREA, THETA, PHI, FZ,
& VAL, dfz, junk, rand
INTEGER M, N, J, L, idum
C
External dfz
V
\[ \pi = 4.0 \times 10^0 \times \text{DATAN}(1.0 \times 10^0) \]

C

OPEN (30, FILE='z.dat', STATUS='NEW', IOSTAT=I)
IF (I.NE.0) THEN
  WRITE (*, '(//1X, A)') 'ERROR OPENING FILE'
ENDIF

C Initializes the random number generator

idum=4
  call srand(idum)
DO L = 1, 250
  junk=rand()
ENDDO

C The surface area of integration

AREA=\pi^2

C Initializes the counter

N = 1000000

C Defines the range and separation of the data points
C for the plots of dFz vs. zo

DO zo= 5.3, 14.05
  C Initializes the sum
  SUM=0.0

  C Performs the summation of function value for one million
  C random points on the surface

  DO J=1, N
    THETA=\pi/2.*RAND()
    PHI=2.*RAND()
    VAL=dFz(THETA, PHI, zo)
    if((.not. (val.ge.0)).and. (.not. (val.lt.0))) VAL=0
    SUM=SUM + VAL
  ENDDO

C Calculates the value of the integral--the surface area times
C the average value of the function over that area

FZ=VOL*SUM/N

C Writes the results to a file as an pair (zo, dFz)
WRITE (30,*) zo, FZ
C
if(fz.ge.0) goto 100
ENDDO
C
100 CLOSE (30,STATUS=’KEEP’,IOSTAT=M)
IF (M.NE.0) THEN
  WRITE (*,‘(/1X,A)’) ’ERROR CLOSING FILE’
ENDIF
END
C
******************************************************************************
C
FUNCTION dfz(THETA, PHI, zo)
******************************************************************************
C
* COMPUTES FORCE OF RAY INCIDENT ON SPHERE AT THETA, PHI *
C
******************************************************************************
C
REAL*8 Pi, Deg, a, ns, nl, m, lambdao, lambda, wo, eo, g, s,
  phi, theta, xo, yo, zo, ro, do, x, y, z, xm, ym, zm,
  rm, b, oml, om2, om3, cm1, cm2, cm3, u1, u2, u3, ang,
  alfa, beta, co1, co2, co3, v1, v2, v3, peri, per2,
  per3, n1, n2, n3, par1, par2, par3, p1, p2, p3, xn,
  yn, zn, rho, w, in, ip, nn, rp, dn, dp, etal, eta2,
  eta3, eta4, un, up, pn, pp, dfz, et
COMPLEX*16 I, q, ex1, ex2, ex3, ey, ez1, ez2, ez, en, ep
C
Pi = 4.0D0*DATAN(1.0D0)
Deg = Pi/180.D0
I = (0.0D0, 1.0D0)
C
C Initializes the variables such as sphere radius, refractive
C index of the sphere and liquid, the wavelength, etc.
C
a = 5.D0
ns = 1.58D0
nl = 1.33D0
m = ns/nl
lambdao = 0.488D0
lambda = lambdao/nl
wo = .33D0
eo = 1.0D0
g = PI*wo**2/lambda
s = lambda/(2.0D0*PI*wo)
C
C Defines the relative position to the beam axis
C
yo = 1.0D-6
xo = 0.0D0
ro = DSQRT(xo**2 + yo**2)
do = DSQRT(ro**2 + zo**2)
C
C Performs the transformation from theta and phi in sphere
C coordinates to (x, y, z) in beam coordinates
x = a*DSin(theta)*DCos(phi)
y = a*DSin(theta)*DSin(phi)
z = a*DCos(theta)
xm = xo*zo*x/(ro*do) + yo*y/ro - xo*z/do + xo
ym = yo*zo*x/(ro*do) - xo*y/ro - yo*z/do + yo
zm = -ro*x/do - zo*z/do + zo
rm = zm + g**2/zh

C Finds the point C
C
b = zm - D.Sqrt(rm**2 - xm**2 - ym**2)
C
C Defines the vectors OM and CM and unit vector u
C
om1 = xm-xo
om2 = ym-yo
om3 = zm-zo
cm1 = xm
cm2 = ym
cm3 = zm-b
u1 = cm1/D.Sqrt(cm1*cm1 + cm2*cm2 + cm3*cm3)
u2 = cm2/D.Sqrt(cm1*cm1 + cm2*cm2 + cm3*cm3)
u3 = cm3/D.Sqrt(cm1*cm1 + cm2*cm2 + cm3*cm3)

C Determines if incidence angle is physically possible
C
IF (- (om1*u1 + om2*u2 + om3*u3)/a .GT. 1.00) THEN
  dfz = 0.00
  GOTO 500
ENDIF
C
C Determines if the incidence angle is less than Pi/2
C If so program continues
C If not program returns the value dfz=0
C
ang = D.ACos(-(om1*u1 + om2*u2 + om3*u3)/a)
IF(ang .LE. Pi/2.0) THEN
  alfa = ang
C
C Calculates the refraction angle beta
C
beta = D.ASin(D.Sin(alfa)/m)
C
C Defines the vectors CO, v, n, and p
C
c01 = xo
c02 = yo
c03 = zo-b
v1 = c01/D.Sqrt(c01*c01 + c02*c02 + c03*c03)
v2 = c02/D.Sqrt(c01*c01 + c02*c02 + c03*c03)
v3 = c03/D.Sqrt(c01*c01 + c02*c02 + c03*c03)
per1 = b*(ym-yo)-ym*zo+yo*zm
per2 = b*(xo-xm)+xm*zo-xo*zm
per3 = xo*ym-xm*yo

viii
\[ n_1 = \text{per1}/\text{DSqrt}(\text{per1} + \text{per2} + \text{per3}) \]
\[ n_2 = \text{per2}/\text{DSqrt}(\text{per1} + \text{per2} + \text{per3}) \]
\[ n_3 = \text{per3}/\text{DSqrt}(\text{per1} + \text{per2} + \text{per3}) \]
\[ \text{par1} = b*(x-m)*b*(2*x-o+z-m) + \]
\[ \text{par2} = b*(x-o)*b*(2*y-o+z-m) - \]
\[ \text{par3} = b*(x+m)*b*(y-o)*z-m \]
\[ \text{pl} = \text{par1}/\text{DSqrt}(\text{par1} + \text{par2} + \text{par3}) \]
\[ \text{p2} = \text{par2}/\text{DSqrt}(\text{par1} + \text{par2} + \text{par3}) \]
\[ \text{p3} = \text{par3}/\text{DSqrt}(\text{par1} + \text{par2} + \text{par3}) \]

C Calculates the electric field vector components

\[ \text{xn} = x/m \]
\[ \text{yn} = y/m \]
\[ \text{zn} = s*z/m \]
\[ \text{rho} = \text{DSqrt}(x^2 + y^2) \]
\[ \text{w} = w_0*\text{DSqrt}(1. + (z/m)^2) \]
\[ \text{q} = g*(z - i*g)/(z^2 + g^2) \]
\[ \text{et} = (e_0*\text{DSqrt}(-x^2 + y^2)/w) \]
\[ \text{ex1} = (s*q)^2*(-rho^2 + i*q*rho^4 - 2.*x^2) \]
\[ \text{ex2} = 8.*rho^2 - 2.*i*q*rho^4 \]
\[ \text{ex3} = (s*q)^4*(2.*rho^4 - 3.*i*q*rho^6 - \]
\[ \text{ex} = (1 + ex1 + ex3)*et \]
\[ \text{ey} = x*n^2*(-2.*s^2 + (s*q)^2 + \]
\[ \text{ez1} = ((s^q)^3*(6.*rho^2 - 2.*i*q*rho^4)) \]
\[ \text{ez2} = (s^q)^5*(-20.*rho^4 + 10.*i*q*rho^6 + \]
\[ \text{ez} = x*n^2*(-2.*s^q + ez1 + ez2)*et \]

C Calculates the components perpendicular and parallel to the scattering plane

\[ \text{en} = \text{ex}*n_1 + \text{ey}*n_2 + \text{ez}*n_3 \]
\[ \text{ep} = \text{ex}*p_1 + \text{ey}*p_2 + \text{ez}*p_3 \]

C Calculates the Intensities, less the proportionality constant

\[ \text{in} = \text{DREAL}(\text{en} + \text{DCONJG}(\text{en})) \]
\[ \text{ip} = \text{DREAL}(\text{ep} + \text{DCONJG}(\text{ep})) \]

C Defines the reflection coefficients

\[ \text{rn} = \text{DSIN}(\text{alfa} - \text{beta})/\text{DSIN}(\text{alfa} + \text{beta}) \]
\[ \text{rp} = \text{DTAN}(\text{alfa} - \text{beta})/\text{DTAN}(\text{alfa} + \text{beta}) \]

C Defines eta and xi
dn = 1. + (2.*rn**2)*DCOS(2.*beta) + rn**4
dp = 1. + (2.*rp**2)*DCOS(2.*beta) + rp**4
eta1 = (DCOS(2.*beta - 2.*alfa) + (rn**2)*DCOS(2.*alfa))/dn
eta2 = (DCOS(2.*beta - 2.*alfa) + (rp**2)*DCOS(2.*alfa))/dp
eta3 = (DSIN(2.*beta - 2.*alfa) - (rn**2)*DSIN(2.*alfa))/dn
eta4 = (DSIN(2.*beta - 2.*alfa) - (rp**2)*DSIN(2.*alfa))/dp

C Defines un, up, pn, and pp

un = 1. + (rn**2)*DCOS(2.*alfa) - ((1. - rn**2)**2)*eta1
up = 1. + (rp**2)*DCOS(2.*alfa) - ((1. - rp**2)**2)*eta2
pn = (-rn**2)*DSIN(2.*alfa) - ((1. - rn**2)**2)*eta3
pp = (-rp**2)*DSIN(2.*alfa) - ((1. - rp**2)**2)*eta4

C Calculates the force element dfz, with the proportionality
C constant

dfz = (28.2235/(wo**2*(1 + 2*g**2 + 5.5*g**4)))*(a**2)*
     DCOS(alfa)*((in*un + ip*up)*u3 +
     (in*pn + ip*pp)*p3)*DABS(DSIN(theta))

ELSE
    dfz = 0.D0
END IF

500 END
References


