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Glory in optical backscattering from air bubbles

Dean S. Langley
College of Saint Benedict/Saint John's University, dlangley@csbsju.edu

Philip L. Marston

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single-mode optical fibers with positive GVD. By using a precise cross-correlation scheme, we measured the output pulse shape from the fiber with 2-psec precision and thereby tested both the predictions and the validity of the nonlinear Schrödinger equation.

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[(a)]Permanent address: Department of Physics, Faculty of Science, Kyoto University, Kyoto 606, Japan.

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[17]The constant $\alpha \sim 1$ adapts these equations to describe propagation in single-mode fibers (Refs. 6, 7, 10, 11, and 14).

[18]The general trend of both the data and the calculations as functions of increasing input power is the following. For input power levels below approximately 0.1 W, the pulse is undistorted by passage through the 70-m fiber. As the power is increased above this value, the pulse broadening, chirping, and self-steepening increase monotonically up to our maximum available input power of 10 W.

Glory in Optical Backscattering from Air Bubbles

Dean S. Langley and Philip L. Marston
Department of Physics, Washington State University, Pullman, Washington 99164
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Observations of light backscattered from air bubbles in a viscous liquid demonstrate an enhancement due to axial focusing. A physical-optics approximation for the cross-polarized scattering correctly describes the spacing of regular features observed. The non-cross polarized scattering is not adequately described by a single class of rays.

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The Mie solution[1] for electromagnetic scattering by a sphere frequently does not lead to direct interpretation of the angular scattering pattern. Consequently, models have been developed to facilitate an understanding of the structure in the scattered intensity present where intensity is plotted as a function of the scattering angle φ or the size parameter $x = ka$ (k is the wave number; a is the sphere radius). These models have emphasized the angular regions where diffraction is important for a drop of water in air: the rainbow,[2,3] $\varphi \simeq 180°$,[3-5] and $\varphi \simeq 0°$.[3,6] In the scattering of light by a spherical air bubble in a liquid or in glass, the real part of the refractive index of the sphere is *less than* that of the surroundings and the models must be significantly modified. New phenomena appear, such as diffraction[7,8] in the region of the critical scattering angle φ_c. Here we report the first detailed observations of backscattering by air bubbles in liquids and give a model which describes some of the observed features. We refer to this as glory because, as in the case of drops,[3-5] the $\varphi \simeq 180°$ scattering is enhanced when x is large.

Van de Hulst[3,4] gave a partial explanation of the enhancement for drops by noting the axial focusing of those backscattered rays which have a nonzero impact parameter. When modeling this focusing in the far field, diffraction provides an essential correction to ray optics because the factor in the scattered intensity which accounts for geometrical divergence of the rays goes to ∞ as $\varphi \to 180°$. Examination of this factor in ray-optics models of scattering by bubbles[9] shows that this infinity is not restricted to drops. We have modeled the backscattering with a physical-optics approximation. The procedure is to (a) compute amplitudes in an exit plane in contact with the bubble via ray optics, and (b) allow this wave to diffract to the far field where the distance from the bubble's center $R \gg ka^2$.

Figure 1 illustrates several rays which lead to backscattering. The paths are determined by the number of chords p and $m = m_i/m_0$, where the refractive indices of the inner and outer media, m_i and m_0, are taken to be real. Figure 1 is drawn with $m^{-1} = 1.403$ which corresponds to an air bubble in the dimethyl-siloxane-polymer liquid used in the experiment. All rays satisfy $\sin\theta = m \sin\rho$. For $\varphi = 180°$, the off-axis (or *glory*) rays have $\theta = \tilde{\theta}$ and $\rho = \tilde{\rho}$, where[10] $\tilde{\theta} = p\tilde{\rho} + (2g + 2 - p)90°$, g is a nonnegative integer ($g = 0$ for rays in Fig. 1), and $m < 1$ requires that $p \geq 3$. The exit plane (dashed line in Fig. 1) touches C' with its normal parallel to the propagation direction of the incident wave.

Our description of the field in the exit plane is facilitated by considering the propagation of a

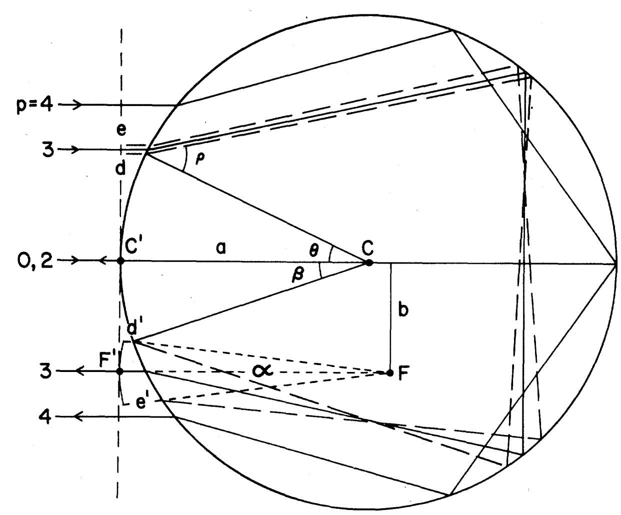

FIG. 1. Rays which contribute to backscattering. The local angle of incidence is θ and C is the bubble's center.

wavelet de which lies close to the backscattered path. Figure 1 shows de for $p = 3$; it emerges as curve $d'e'$. This curve appears to come from a ringlike source at F known as the *focal circle* in the analogous $p = 2$ scattering from drops[4] with $\sqrt{2} < m < 2$. The source is ringlike because the figure may be rotated around the CC' axis. The radius of the ring is $b = a \sin\tilde{\theta}$. After the incident ray crosses the dashed vertical plane (the entrance plane), the *propagation* phase delay for reaching the exit plane is $\eta = ka[1 - \cos\theta + (1 - \cos\beta)\sec(\theta - \beta) + 2mp\cos\rho]$. The ray crosses the exit plane at a radius s from C' with $s/a = \sin\beta - (1 - \cos\beta)\tan(\theta - \beta)$. The radius α of arc $d'e'$ follows from the curvature at $s = b$: $\alpha = k(d^2\eta/ds^2)^{-1} = a[1 + \frac{1}{2}(p\tau - 1)^{-1}\cos\tilde{\theta}]$, where $\tau = \tan\tilde{\rho}/\tan\tilde{\theta}$. The spreading of the wavelet is characterized by $q = \lim(\overline{d'e'}/\overline{de})$ as $\overline{de} \to 0$ where the bar denotes the arc length. An equivalent expression for q is $|\lim[b - s(\theta)]/(b - a\sin\theta)|$ as $\theta \to \tilde{\theta}$; its value from L'Hospital's rule is $\alpha/(\alpha - a)$. Vectors $\hat{e}_l$ ($l = 1, 2$) denote orthogonal basis vectors in both the entrance and the exit planes; $\hat{e}_1$ is chosen parallel to the polarization of the incident wave's electric field $E_1 \exp(-i\omega t)$.

In the exit plane, the field $E_p{}^l \hat{e}_l$ of the outgoing pth glory wave is computed by applying Van de Hulst's method of first decomposing the fields perpendicular and parallel to the scattering plane.[3,4] Exit-plane polar coordinates centered on C' are (s, ψ), where ψ is the angle relative to $\hat{e}_1$ and $\hat{s}$ and $\hat{\psi}$ denote local basis vectors. We assume $x \gg 1$ and use Fresnel's coefficients r_j for the internal reflections where $j = 1, 2$ for fields parallel to $\hat{\psi}$ and $\hat{s}$, respectively. If $|s - b| \ll a$, the multiple internal reflections give

$$E_p{}^l = E_I q^{-1/2} F^l \exp[i\tilde{\eta} + ik(s - b)^2/2\alpha], \qquad (1)$$

where $\tilde{\eta} = \mu + \eta(\rho = \tilde{\rho})$, $F^1(\psi) = c_1 \sin^2\psi + c_2 \cos^2\psi$, $F^2(\psi) = \frac{1}{2}(c_2 - c_1)\sin 2\psi$, and $c_j = (-1)^{p(j-1)} r_j{}^{p-1}(1 - r_j{}^2)$. The new phase term μ accounts for the crossing of caustics or "focal lines"; its value is[3,11] $-\pi(p + g)/2$. The r_j are evaluated at $\tilde{\theta}$: $r_1 = \sin(\tilde{\theta} - \tilde{\rho})/\sin(\tilde{\theta} + \tilde{\rho})$ and $r_2 = \tan(\tilde{\theta} - \tilde{\rho})/\tan(\tilde{\theta} + \tilde{\rho})$. The sign factor in c_j accounts for a geometrical inversion (present when $j = 2$ and p is odd) which is not evident in descriptions of $p = 2$ glory in drops.[3,4]

The field $E_p{}^l$ at a distant point Q is computed as follows. The left extension of the CC' axis makes an angle γ with $C'Q$. When γ is small and $\overline{C'Q} = R' \gg ka^2$, scalar diffraction theory and the Fraunhofer approximation[12] give

$$E_p{}^l \simeq \frac{kE_I \exp[i(kR' + \tilde{\eta})]}{2\pi i R' q^{1/2}} \times \int_0^\infty s W^l \exp[ik(s-b)^2/2\alpha]\, ds, \qquad (2)$$

$$W^l = \int_0^{2\pi} F^l \exp[-iks \sin\gamma \cos(\psi - \xi)]\, d\psi, \qquad (3)$$

where ξ is the angle between $\hat{e}_1$ and the projection of $\overline{C'Q}$ on the exit plane. In Eq. (2), the approximation given by Eq. (1) has been extended beyond its useful domain in anticipation of the stationary phase approximation (SPA) of the integral. Direct evaluation of Eq. (3) gives $\tilde{W}^1(\gamma,\xi) = W^1(\gamma,\xi,s=b) = \pi[(c_1+c_2)J_0(u) + (c_1-c_2)J_2(u)\cos 2\xi]$ and $\tilde{W}^2 = \pi(c_1 - c_2)J_2(u)\sin 2\xi$, where $u = kb\sin\gamma$. The SPA of Eq. (2) gives the pth glory contribution to the scattered field when kb^2/α, and thus x, are large. In the experiments to be described $x \geq 4000$ and the SPA is applicable.

The total field may be approximated by summing the $E_p{}^l$ from Eq. (2) with the fields due to axial reflections and surface waves. Surface wave contributions should be small for the observed bubbles because of the largeness of x. To determine which glory and axial terms are important to the total field, and for other heuristic reasons, consider the l-polarized intensity $I_p{}^l$ of the pth field *taken alone.* The SPA of Eq. (2) gives

$$I_p{}^l = (2/\pi) x I_R f_{p,g} [\tilde{W}^l(\gamma,\xi)]^2, \qquad (4)$$

where $I_R = I_I a^2/4R^2$ is the total intensity at a distance $R = \overline{CQ}$ from a *perfectly reflecting sphere* of radius a predicted by *ray optics*,[9] I_I is the incident intensity, and $f_{p,g} \equiv b^2\alpha/a^3 q = b^2(\alpha - a)/a^3$. In Eq. (4), R has replaced R' from (2) and γ becomes $180° - \varphi$ because $R \gg a$. Geometrical optics[3,9] gives the intensities $\check{I}_p{}^l$ of separate axial reflections (e.g., $p=0$ and 2 in Fig. 1) which are proportional to a^2. The strongest reflection has $p=0$ and $l=1$; for $\gamma=0$, $\check{I}_0{}^1 = I_R(m-1)^2/(m+1)^2$ while $\check{I}_0{}^2 = 0$. Since $f_{p,g}$ does not depend on a, $I_p{}^l \propto ka^3$ and glory terms dominate the backscattering when a is large.

Consider a bubble with $x = 4000$ and $m = 1.403^{-1}$. The strongest glory terms have $g=0$ and $p=3$, 4, and 5; the $I_p{}^1/I_R$ for $\gamma=0$ are, respectively, 1.03, 0.43, and 0.16. The $I_p{}^1$ decrease with increasing p as a result of the partial reflections in the bubble. The strongest axial ray gives $\check{I}_0{}^1/I_R = 0.028$. The interference of the fields depends on a and our Mie computations verify that the backscattered intensity is not simply proportional to a^3 even for this large value of x. The $l=2$ (cross-polarized) scattering is, however, nearly dominated by the $p=3$ glory term. Because of symmetry, $l=2$ scattering vanishes as $\gamma \to 0$. The $I_p{}^2(\gamma \neq 0, \xi)$ have maxima at $\xi = \pm 45°$ and $\pm 135°$ and they vanish at $\xi = 0°$, $\pm 90°$, and 180°. Let $\gamma = \gamma_p$ locate the first maxima of $I_p{}^2(\gamma, \xi = 45°)$. The largest $l=2$ terms have $I_p{}^2(\gamma_p, \xi = 45°)/I_R = 0.53$ and 0.10 for $p=3$ and 4. To the extent that $p \neq 3$ scattering may be neglected, the $l=2$ intensity will be quasiperiodic in γ.

We have numerically verified the validity of Eq. (4) by using Debye's localization principle[3,4] to modify Mie theory so that only partial waves associated with $p=3$ rays were included in the Mie series. Furthermore, when Eq. (4) is applied to spheres with certain $m > 1$, the resulting $I_p{}^1(\gamma=0)$ agree with the glory "analog" tabulated in Ref. 11. This analog was derived by applying the Watson transformation to the $\gamma=0$ Mie series.

Figure 2 diagrams the experiment. A syringe injected bubbles into the liquid. The liquid had a high kinematic viscosity [$\simeq$ 600 000 cS; 1 stoke (S)

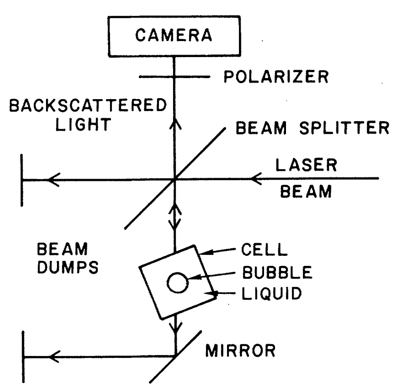

FIG. 2. Apparatus for observing backscattering from bubbles.

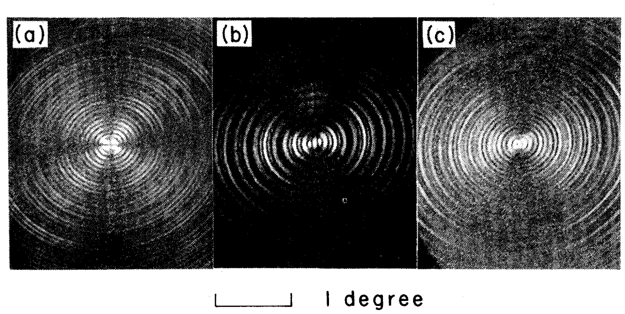

FIG. 3. Photographs for (a) crossed polarizer ($l = 2$ scattering); (b) uncrossed polarizer ($l = 1$); and (c) no polarizer. The incident polarization was vertical. $a = 0.49$ mm and $x = 6830$.

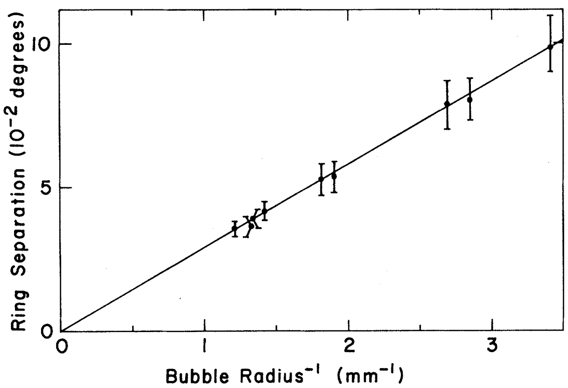

FIG. 4. Measurements and model for the angular separation of the dark rings in the $l = 2$ scattering.

$=1$ cm^2/sec] and a single bubble could be observed for hours. The laser's power output was 5 mW and the beam diameter was 5mm. The wavelength in the liquid, $2\pi/k$, was (632.8 nm)/1.403; $\hat{e}_1$ lay in the splitter's plane of incidence. The camera was focused on ∞ so that the photographs recorded the far-field intensity pattern.[7,12] Photographs were made with $a \simeq 0.3-0.8$ mm corresponding to $x \simeq 4000-11\,000$. Exposure times were typically 5 s for TriX film and a 200-mm–focal-length camera lens.

Figure 3 demonstrates that the scattering has roughly the dependence on ξ predicted by Eq. (4); $\xi = 0°$ corresponds to scattering toward the top of the photographs and $\gamma = 0°$ corresponds to the center of the symmetry. Figure 3(b) shows that the $l = 1$ scattering for $\gamma > 0.2°$ is *significantly stronger* for $\xi = \pm 90°$ than it is for $\xi = 0°$. This agrees with the following model results: (i) $(c_1/c_2)^2 \gg 1$ (for $p = 3$ we predict $c_1/c_2 \simeq -5.2$); and (ii) for this x, the $\check{I}_p^1$ depend only weakly on ξ and are dominated by the I_p^1. One prediction of Eq. (4) could be quantitatively checked: when both $\sin\gamma \simeq \gamma$ and $u \gg 1$, the minima in I_p^2 should be spaced by $\Delta\gamma$ rad such that $kb\Delta\gamma \simeq \pi$, where for $p = 3$, $b/a = 0.447$. Figure 4 compares this with the mean spacing of ≈ 40 dark rings lying outside the 9th ring from the center. The error bars combine uncertainties in measured a and $\Delta\gamma$ with those of corrections due to refraction at the cell-air interface[7] and the tilt of the cell. Figure 4 shows that $p = 3$ rays dominate the $l = 2$ scattering. The modulations of the intensity along $\xi = \pm 45°$ in Fig. 3(b) show that other rays contribute to $l = 1$ scattering since the predicted $I_3^1 \propto [J_0(u)]^2$.

In conclusion, backscattering from bubbles can be enhanced by axial focusing. The number of significant glory terms depends on m. The main contributions differ from those for water drops where surface waves[3] and other diffraction related terms[5] play an essential role. If focusing were not present, scattering by large bubbles would be $\ll I_R$ in the region[7-9] $(\varphi_c + 10°) \lesssim \varphi \lesssim 180°$, where $\varphi_c = 2\cos^{-1} m \simeq 89°$ for $m^{-1} = 1.403$. We also find evidence of $p = 3$ glory in Mie computations for bubbles in *water*.

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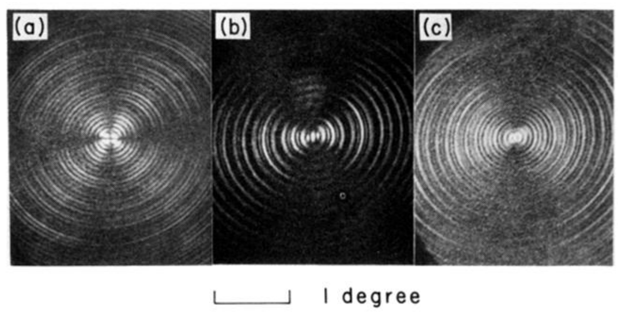

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