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How Long Does Savings Last When Retirees Need More Money (or Less) than Theory Suggests?

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Abstract

Popular convention is to initially withdraw approximately 4% of the retirement savings and increase that dollar amount each year by inflation. But, what if 4% isn't enough? How long will retirement funds last if a newly retired person needs 5%, 6%, or even 10%? Modern Portfolio Theory (MPT) suggests that an investor only needs to choose between 2 assets, the risk free rate and an optimal risky portfolio. In this paper, five U.S.-based assets are tracked from 1934 until 2015 to see how long they survived independently and in combination with one other asset. Obviously the more a person needs to withdraw from retirement assets, the more quickly the assets are depleted. This paper shows just how fast (or slow) that happens. The risk of the portfolios is also assessed and addressed.

INTRODUCTION

There is a plethora of information written about when to retire, how much money one needs in order to retire, and how much return one needs and risk one should take in his or her portfolio. To an individual who is on the precipice of retirement, most of these decisions are in hindsight, though. The decisions facing this person for the future are primarily how much to withdraw and how much risk to take with previously accumulated wealth in retirement.

Once a person has saved up a nest egg and made the decision that it is time to retire and live off of savings, it is time to take stock of investment assets in retirement accounts and assess how long that money will last. The popular convention is to initially withdraw approximately 4% of the retirement savings and increase that dollar amount each year by inflation (See Bergen, 1994, Rosenthal, 2013, or Waggoner, 2013, for example). Some studies use a long-term average inflation rate for simplification (Israelsen, 2016), but it is easy enough to adjust income by the prior year's inflation since CPI data for the prior year is usually available by about January 20 of each year.

First, I gathered data from 5 U.S.-based asset classes: small cap stocks, mid-cap stocks, large cap stocks, a 10-year constant maturity Treasury bonds, and 3-month T-bills from 1934 to 2015. Then, I discuss the implications of investing in these portfolios along with the inherent riskiness of each using the Sharpe ratio as my primary risk measure.

The core of the analysis follows as I looked at initial withdrawal rates from 2% to 10% (increasing annually after that with inflation) over rolling periods of 5, 10, 15, 20, 25, 30, 35, and 40 years to see what history tells us is the likelihood that our retirement funds will survive at least that length of time. I allowed the retiree to select one or two asset classes in 10% increments and rebalanced them each year. This was to allow for several "stock-only" choices along with several choices that include short-term and long term risk-free rates. There are 95 distinct portfolios.

Results indicate that the all-stock portfolios are far more likely to survive in the longer time frames, albeit with more volatility risk. If a retiree needs to withdraw higher amounts than is conventionally recommended, then higher risk is required in order to allow the opportunity for the portfolio to survive as long as possible. Although, all portfolios at all levels survived for five years, at the 10% initial withdrawal rate, the T-bond portfolios and T-bill portfolios had very little chance of surviving for even fifteen years without a significant stock counterpart. On the other hand, nearly one-half of some combinations of all-stock portfolios survived for 30 – 35 years.

If looking at the portfolios strictly in terms of risk, the portfolio with the best single-asset or 2-asset portfolio Sharpe ratio is composed of 30% small cap stocks and 70% Treasury bonds. While the risk-return relationship was best, this portfolio combination had nearly no chance to survive beyond 15 years if the initial withdrawal rate was 10%. Lower withdrawal rates fared much better with the highest Sharpe ratios.

Based upon the results, there are numerous implications for real-world investors at or near retirement. The key for those nearing retirement will be balancing the volatility risk with the

longevity risk. Practically, that means a retiree must select a risk-return combination that will survive as long as possible, given his or her withdrawal needs.

The rest of the paper is divided into the following sections: The motivation for the paper is discussed followed by data and methodology sections. Then the results and analysis are looked at followed by some recommendations and the conclusion.

MOTIVATION FOR THIS STUDY

Retirement is a large and growing field with approximately 10,000 new retirees each day (Bernard, 2016). Retirees have a different perspective on investments than many people in younger generations. They must be concerned with how much money they have *and* how long it will last. Figuring out how much to withdraw from retirement accounts can be a stressful experience.

Retirees have a shorter time horizon and therefore rightly view risk differently than other age groups. They also approach risk in a different way than is traditionally examined by academia. Retirees have two very important risks to consider and balance. First is the traditional risk we associate with the volatility of an investment. When investing in stocks and bonds, we often consider how "risky" or "conservative" an investment is by what percentage is invested in stocks and what percentage is invested in bonds. This is because stocks have a greater chance of having negative returns, and sometimes significantly negative returns. On the other hand, bonds rarely have large negative returns.

Another risk that is incredibly important to many retirees and is often overlooked by academia is the risk of how long their savings will last in retirement. A reasonable assumption this paper makes is that most retirees prefer not to outlive their money. In attempting to achieve that goal, assessing how much may be withdrawn each year from their retirement account is not a trivial task. One of the most commonly cited ideas is to withdraw 4% of retirement funds in the first year of retirement and increase that amount by inflation each year. It is said that 4% is the "highest rate that held up over a period of at least 30 years." (Bengen, 1994)

This paper deals with some problems with this rigid rule. What if you retire early? What will hold up for 40 years? What if you retire late or have a chronic illness and don't care about 30 years? What if 4% just isn't enough for you to live in retirement or you might be expecting money later and want to spend more now? There are many other permutations, but it is clear that the 4% rule is inflexible to people's needs.

In addition, some are reconsidering the "4% standard" given the low interest-rate environment in which we find ourselves. (Updegrave, 2016) The idea is that the withdrawal rate should be *lower* than 4% if we want our funds to last at least 30 years in retirement, especially since retirees typically shift funds into safer accounts as they get older (e.g. bonds, CD, savings accounts, etc.).

Although Harry Markowitz's Modern Portfolio Theory (MPT) (Markowitz) has come under some scrutiny in recent years because of several assumptions that do not conform to reality and how it holds up in financial crises, it is still a very solid theory upon which to base investment decisions (Omisore et al., 2012). Specifically, the implication of MPT that one can have a

complete portfolio with only two assets, a risk-free asset and a risky portfolio, may be used to create portfolios that are acceptable to investors with different levels of risk aversion and differing needs in retirement.

The goal of this paper is to give the reader an idea of how long retirement savings will last given withdrawal rates from 2% to 10%. Further, the assets may be invested in any one or two of five asset classes at 10% increments. The portfolios used are historical results of how the markets actually fared and would have affected real portfolios. It was felt that this might give a better idea of how long savings will actually survive than by assuming returns and risk are normally distributed over time.

DATA

Risk-free rate returns were extracted from the Federal Reserve Economic Data (FRED) interest rate database which is compiled by the Research Division of the Federal Reserve Bank of St. Louis. For the short-term risk free rate, I annualized the *3-month Treasury bill: Secondary market rate* which starts in 1934. FRED only has data back to 1953 for the 10-year constant maturity Treasury bond. However, data was compiled using FRED by Aswath Damodaran that goes back to 1928 (Damodaran, 2016). Since current data were only available directly from FRED for Treasury bills from 1934, this was used as the start date for the analysis in this paper.

To match available date ranges above, data was collected for stock returns from 1934-2015 based upon size (univariate) using the Kenneth French Data Library. The data represents the largest 30%, the middle 40%, and the smallest 30% of stocks based upon market capitalization and are reconstructed at the end of each June. All NYSE, AMEX, and NASDAQ stocks with a positive market equity are included. Returns are equally-weighted returns (with dividends), annualized from January to December.

Inflation was derived from the Consumer Price Index – All Urban Consumers, All Items (CPI-U), and retrieved directly from the Bureau of Labor and Statistics (BLS).

Of note, the data does incorporate a significant portion of the Great Depression, but does not include the market crash of 1929. This was not intentional. The only limiting factor was the availability of the Treasury data from FRED. The starting year, 1934 and the subsequent two years were great years for the U.S. stock market (followed by a terrible year). So, there may be an upward bias to the data based upon the particular start date. However, that could be said for any particular start date that was hand-picked. There would be a potential upward or downward bias to longer-term results. For example, if starting with 1937, small caps, midcaps and large caps had respective returns of –53%, –47%, and –37%. It may be that bonds outperform stocks for a significant period after this starting date. This is also a problem with retirement. Retirees do not get to choose when they turn 65 years old. Their choices of retirement dates is often relatively limited due simply to their date of birth (i.e., the data they were given).

METHODOLOGY

The methodology for this study was labor-intensive, but not very complex. After downloading and separating the data into five assets: large cap stock, midcap stock, small cap stock, 10-year T-bonds, and 3-month T-bills; I created one- and two-asset portfolios including all combinations that comprised from 0% to 100% of each asset, in 10% increments. There are 95 distinct portfolios, five of which are single asset portfolios (i.e., 100% in the asset).

Starting on January 1, 1934, I assume a withdrawal amount from 2 – 10%. This amount then becomes the base withdrawal amount which is increased by inflation each year. I used the December to December CPI-U to represent inflation.

For each of the 95 portfolios, I discover how long the portfolio remains solvent (survives) at each of the withdrawal rates. For simplicity, I divide portfolio lives into 5-year increments. I check each 5-year increment from 5 years to 40 years. I did make the assumption that 40 years of retirement may be the longest term most people would consider. For 40 years of retirement a person who retires at 65 would have to live until he or she was 105 years old, well beyond a normal life expectancy. However, reasonably, I think a 40-year retirement window might be relevant if a person retired in his or her early 50s or younger. There were 78 rolling five-year windows in the data from 1934 – 2015. There were 73, 68, 63, 58, 53, 48, and 43 rolling windows of 10, 15, 20, 25, 30, 35 and 40 years, respectively. In the analysis, I have 855 portfolios, 95 each at 2%, 3%, 4%, 5%, 6%, 7%, 8%, 9%, and 10% withdrawal rates. The percentages of each portfolio surviving for the lengths described above were tabulated.

To illustrate a simple example, there are 63 rolling 20-year periods from 1934-2015. Therefore, there are 63 opportunities for a given portfolio to survive or fail during any given 20-year window. At the 8% withdrawal rate, and using a portfolio with 60% midcap stocks and 40% T-bonds, the portfolio survives for 38 of the 63 20-year periods, yielding a 60% survival rate (38/63 = 0.603).

I looked at each portfolio's long-term expected return, standard deviation, and Sharpe ratio for the entire 82-year period. Of course, while we may expect a particular return for a riskier portfolio, for any given year the portfolio may lose money. For the Sharpe ratio, I used T-bills for the risk-free rate, realizing that I also have T-bonds in the set of asset classes. T-bonds could be used as well, but that would create a negative expected Sharpe ratio when using T-bills and T-bonds in the same portfolio. T-bonds would likely be more appropriate than T-bills for horizons of 10 years or longer. For the sake of consistency and less confusion, I decided to keep the same risk-free rate for all portfolios rather than switch from T-bonds to T-bills as the horizons got shorter.

Portfolios are assumed to be rebalanced on January 1 of each year. And, I should note that taxes are not considered in this analysis. There are so many possible tax ramifications with all of the various retirement plans that it is clear that I could not do justice to the topic in this paper.

RESULTS & ANALYSIS

Table 1 shows the correlation matrix of the assets. The assets were correlated in the way one might expect. Stocks were very highly correlated with each other, but not with the bonds or bills. In fact, all stock portfolios were at least weakly negatively correlated with both bonds and bills. The bonds and bills were positively correlated with each other, but not strongly.

Table 1. Correlation Matrix among Assets

	SmCap	*MidCap*	*LgCap*	*10yTBond*	*TBill(Cash)*
SmCap	1.0000				
MidCap	0.9309	1.0000			
LgCap	0.8081	0.9467	1.0000		
10yTBond	-0.1978	-0.1143	-0.0557	1.0000	
TBill(Cash)	-0.1413	-0.0901	-0.0470	0.2864	1.0000

A Simple Risk-Reward Perspective

There are many ways to look at the data. A simple description of expected returns over the 82-year period shows exactly what one might expect: small caps have the highest expected return, followed by midcaps, large caps, T-bonds, and T-bills. Some descriptive statistics for the full period are in Table 2.

Table 2. Some Descriptive Statistics for Asset Classes from 1934-2015

ASSET	E(R)	SD	CV	SHARPE
SMALLCAP	20.47%	34.99%	1.7091	0.4833
MIDCAP	15.46%	24.11%	1.5591	0.4936
LARGECAP	13.01%	19.20%	1.4751	0.4923
TBONDS	5.40%	8.23%	1.5238	0.2233
TBILLS	3.56%	3.15%	0.8837	0.0000

Traditionally, risk is often compared to return using the Sharpe ratio. Of the 95 portfolios, 7 had Sharpe ratios >0.5500, 20 portfolios had Sharpe ratios between 0.5000 and 0.5500. The majority of portfolios (50) had Sharpe ratios between 0.4500 and 0.500. The remaining 18 portfolios had Sharpe ratios below 0.4500, including all 11 portfolio combinations that included T-bonds and T-bills.

The highest Sharpe ratio among the 95 portfolios was 0.5818 which consisted of 30% small cap stocks and 70% T-bonds and had an absolute expected return of 9.92%. The portfolio seems reasonably conservative, though with a large portion in bonds. Interestingly, this portfolio survived for 40 years 100% of the time for 2% and 3% withdrawal rates, but had a much more difficult time surviving as the withdrawal rates increased. The 4% withdrawal rate was still good with 95% of portfolios surviving 40 years. But, if you need to withdraw more than 4% from your retirement savings, the survival rates drop precipitously. At the 5% withdrawal rate, 89% of the portfolios survive for 30 years, but only 56% survive for 40 years. At the 6% withdrawal rate, 89% of the portfolios survive for 20 years, but only 43% survive for 30 years, and a dismal 14%

survive for the 40-year retirement span. The moral here is that if you want the best risk-return ratio, you probably should not take out more than 4% initially if you expect to live 40 years into retirement. If you need 5%, you should wait to retire until your life expectancy is less than 30 years, and if you need 6%, you should wait to retire until you only expect to need the savings for 20 years. Any withdrawal amount above 6% gets much worse unless your time horizon is 10 years or less.

Other asset combinations that compared relatively well to the highest Sharpe ratio were combinations that included 30-60% stock and 70-40% T-bonds. All of these combinations had Sharpe ratios above 0.5200. No combinations using T-bills ever had a Sharpe ratio above 0.5000.

If we strictly look at the Modern Portfolio Theory and use combinations with either T-bonds or T-bills as one (risk-free) asset and stocks as the other (risky) asset, we may limit ourselves in risk-return and in total expected return. Over the long-run that means we risk running out of money as we get into the later years of retirement. This is why small cap/midcap, small cap/large cap, and midcap/large cap combinations were also considered.

A More Realistic "How Much Do I Really Need?" Perspective

When we look more directly at how long a retiree's money is likely to last, it might be better to answer the question: How much money does the retiree *need* to withdraw from his or her retirement account in retirement? From that answer, we can estimate how risky the retiree might need to be with his or her savings in retirement in order for the money to survive as long as its owner.

If a retiree needs to withdraw only 2% of retirement savings and increase that amount by inflation each year, this person is safe (and lucky). Assuming that the past 82 years are indicative of future results, his or her retirement savings will last at least 30 years under all scenarios. As long as the money is invested in one or more of the financial assets described in this paper, the only chance that the money will not last 40 years, is if it is all invested in T-bonds and/or T-bills. Even 10% in stock and 90% in T-bills last for 40 years.

If the retiree needs to withdraw 3% of retirement savings, all scenarios with at least 10% stock will last at least 25 years. Also, of the possible combinations that include at least 10% stock, there is a 98% chance that the portfolio will last 30 years, a 93% chance it will last 35 years, and an 89% chance it will last for 40 years. Further, there are many other portfolio combinations that survived for 40 years 100% of the time. If you only need 2-3% of your retirement savings, you will likely not have a problem running out of money in retirement unless future conditions are drastically different (and worse) than conditions in the past.

Bengen (1994) and many subsequent researchers have declared that 4% is a relatively safe amount to initially withdraw from a retirement account. Increasing withdrawals by inflation each year will still give the retiree some certainty that his or her money will last for most, if not all of his or her retirement. As long as one does not put almost all of his or her money in T-bonds or T-bills, this advice seems to reflect reality. There are four portfolio combinations that survived for 30 years 100% of the time. However, there is a small chance that even with the best combinations a retiree might run out of money if his or her horizon is 35 or 40 years. Most

portfolios consisting of T-bonds and T-bills only survived for 25 years about 50% of the time, though. So, retirees must include some stocks in the retirement portfolio throughout the retirement years if he or she wants a withdrawal rate of 4% and expects the portfolio to last beyond 20 – 25 years. The four portfolios that always survived for 30 years consisted of 30-40% small cap stocks and 70-60% T-bills or 40-50% midcap stocks and 60-50% T-bills.

Things quickly get more complex when encroaching upon withdrawal rates of 5% or more. If the retiree needs a 5% withdrawal rate, his or her money will last 15 years 100% of the time in 39 of the 95 combinations. However, there are no combinations that have a 100% success rate at the 20-year mark. However, if prudently including stock in the portfolio, the vast majority of combinations will have at least a 90% chance of making it to 20 years. Further, if choosing a portfolio combination wisely, there are several that have a 90% or greater chance to survive for 40 years. Many of these are all-stock portfolios, but a few have significant portions of bonds or bills. The portfolio combination that did the best was 70% small cap stocks and 30% T-bonds. This portfolio combination had a 95% survival rate.

If the retiree needs to withdraw 6% from retirement savings, we will often find that savings does not last beyond 20 years if investing solely in bonds. There are many stock-bond combinations that survive for 20 years, though. For the 20-year time horizon, 60% small cap stock and 40% T-bonds had the highest survival rate (97%). For the 25-year time horizon, two combinations had a 95% survival rate: 80% small cap stocks/20% T-bonds and 90% small cap stocks/10% T-bills. For the 30-year time horizon, a retiree should be 90% in small cap stocks. The other 10% could be in either T-bonds or T-bills. These combinations survived 93% of the time. For the 35-year horizon, 90% small cap stock and 10% T-bonds fared best. That combination survived 92% of the time. And, for the 40-year time horizon, there were five combinations that survived 88% of the time: 100% small cap stocks and 80-90% small cap stocks with 20-10% T-bonds or T-bills.

For the 7% withdrawal rate, we might start thinking about a later retirement. None of the combinations succeed more than 83% of the time for a 30-year time horizon. That leaves about a one in six chance of running out of money. However, if delaying retirement is not an option, we can at least choose the best combination for our time horizon. For 5-10 year horizons, there are many options that survive 100% of the time. For 15 years, the asset combination that included 80% small cap and 20% invested in either T-bonds or T-bills survived 96% of the time. For the 20-year horizon, 5 combinations survived 89% of the time. All 5 combinations are all-stock. Bonds and bills never survived as long in the 20-year period. The 5 combinations all included small cap stocks. 60-80% small cap stocks combined with 40-20% large cap stocks and 60-70% small caps paired with 40-30% midcap stocks survived the longest. For the 25-, 30-, 35-, and 40-year horizons there were multiple pairs of assets that survived the longest. But, one particular combination was at the top of all time horizons of 25 years or greater. It was 70% small cap stocks and 30% T-bonds. This paring survived about 79-86% of the time, depending upon the length of the time horizon.

If an 8% withdrawal rate is desired, the 10-year time horizon became less certain. Only two combinations of assets survived 100% of the time: 30-40% large cap stocks combined with 70-60% T-bills. Of note, there was no specific combination of assets that yielded the best survival rates across several time horizons. Several asset combinations had an 88% success rate at 15 years, some included T-bonds. The same may be said of the 20-year horizon, but the pairings were different. Five pairings had an 86% success rate, two of which included either T-bonds or

T-bills, while the other three were all-stock portfolios. The commonality was that all five included small cap stocks. For the 25-year horizon, the asset combination that included 80% small cap stocks and 20% large cap stocks had the best survival rate (83%). There were three portfolios that survived 77% of the time at the 30-year time horizon. All included 90% small cap stocks. The other 10% could be in any of the assets except T-bills. Interestingly, the 30-year time horizon had worse results than the 35- and 40-year horizons when looking solely at the longest surviving asset combinations. Only 77% of portfolios survived in the best combinations whereas 79% of the portfolios survived in the best combinations at the 35- and 40-year time horizons. For the longest horizons (35- and 40-years), a 100% small cap stock portfolio survived 79% of the time. One other portfolio survived as long at the 35-year horizon. It included 90% small cap stocks and 10% T-bills.

When considering a 9% withdrawal rate, we seem to be on shaky ground. If withdrawing 9% is necessary at retirement, portfolios may start to fail at around only 10 years into retirement. The best combinations to survive for at least 10 years involve allocations of 30-50% small cap stocks with 70-50% T-bills. Those combinations survived for 10 years, 95% of the time. For a 15-year horizon, the best combinations survived 85% of the time and always included a large portion of small cap stocks combined with midcaps, large caps, or T-bonds. For the 20-year horizon and longer, we would need to abandon bonds to try to make the savings last as long as possible. The maximum success rate was 79% using 70% small caps and 30% large cap stocks for the 20-year time horizon. Only 64 – 71% of portfolios survived for 25 – 40 years with the best combinations. These all included either 90% or 100% small cap stocks with the remainder being in midcaps or large caps.

When examining the 10% withdrawal rate, the most aggressive (or drastic) rate considered in this paper, survival rates are often dismal. However, they are still reasonable for a 10-year time horizon, and possibly acceptable at the 15-year horizon, depending on risk aversion levels. However, beyond that, one would likely run out of money or be required to severely curtail withdrawals. Almost all asset combinations survive for five years. The worst choices have a 97% survival rate. For the 10-year horizon, several asset combinations survive more than 80% of the time. The best rate is an 89% survival rate which is achieved with four portfolios that include a 70-40% small cap stock allocation and 30-60% invested in T-bonds. For 15-, 20-, and 25- year time horizons, the best survival rates are either 100% in small cap stocks or 90% small caps and 10% large caps. The percentages associated with those choices drop precipitously as the horizon increases, though. For the 15-year time horizon, the best choice survives 79% of the time which may be acceptable to a person who needs the money and is less concerned about running out of funds in his or her last years of retirement. For 20- and 25-year horizons, the best survival rates are 65% and 55%, respectively. It is unlikely that a typical financial representative would recommend such an investment due to the unacceptably large possibility of running out of money before the estimated time horizon elapses. For the 30-, 35-, and 40-year horizons, the best survival rates are between 45% and 50%. So, even when intentionally selecting a course of action that is likely to survive the longest, the actual survival chances are no better than a coin flip. Of note, at the initial 10% withdrawal rate, T-bills never survived longer than 11 years and T-bonds never made it past 21 years. Each of the all-stock portfolios had at least one 40-year rolling period that survived for the entire 40-years.

SOME RECOMMENDATIONS

There are a few common themes in the results presented above.

If a retiree needs only 2-4% withdrawal rates, he or she is fine sticking with the theories and recommendations prescribed by many financial advisors which is to keep some money in stocks, but to keep a significant portion of assets in bonds as well since bonds tend to be less volatile, and you cannot afford to lose a lot of money in your retirement savings account as you move through retirement. For these withdrawal rates, if 40% or more is held in stocks, there is a 90% survival rate or more with most asset combinations that include both stocks and bonds even for 35- and 40-year retirement time horizons.

As required withdrawal rates increase, there are fewer good options that include large portions of bonds. For 5-6% withdrawal rates, a 90% survival rate is certainly possible in the shorter time horizons. For the longest time horizons, the retiree must select his or her portfolio carefully, assume a higher (volatility) risk tolerance, and prepare for a greater chance of the portfolio not surviving for the duration expected.

For any of the highest withdrawal rates (7-10%), a retiree should only be in all-bond and bill combinations if he or she has a short time horizon, certainly no more than 15 years, and that may be too long. To make the retirement savings last as long as possible for the longer time horizons, the retiree should be significantly in small cap stocks (80-100%) and possibly some midcap or large cap stocks (20% or less).

Small cap stocks seem to be an important asset to consider, *even throughout retirement*, if a retiree would like his or her retirement assets to survive for extended periods of time, especially as withdrawal rates increase. However, as with any investment, one must understand what is meant in this paper by "small cap" stocks. As mentioned, I used data from the Kenneth French Data Library. In that data, small cap stocks are defined as the smallest 30% of stocks listed on the NYSE, AMEX and NASDAQ stock markets. Stocks with no data or negative equity were excluded. What this means to investors is that this is a well-diversified small cap/microcap index. An investable asset that might substitute for this index would be a small cap ETF combined with a microcap ETF.

In addition to worrying about how much money to take out of their retirement portfolios, retirees should keep risk in mind when investing and only bear as much as they may desire. As noted, the best risk-return portfolio included 30% small cap stocks and 70% T-bonds. This is a good, safe option for many retirees. However, the more money the retiree is required to withdraw, the more he or she will have to balance the risk of investment volatility with the risk of running out of money too soon. Ultimately, the risk that wins out will play a significant role in how long retirement savings will survive.

CONCLUSION

This paper is meant to be a challenge to the simple, but rigid rule of withdrawing 4% of retirement savings in the first year of retirement, then increasing that by inflation each year and

hoping the money survives the retiree. Many retirees need amounts above 4% to pay for everyday needs and expenses.

Since there are so many variations and each retirement situation presents circumstances that are unique, or nearly so, it is impossible to cover all possible variations and give accurate recommendations that will be sweepingly accepted by even a majority of retirees. However, by looking at what types of portfolios have actually survived over the past 82 years, we may offer a retiree an idea of how to invest his or her savings so that the money will last for the period desired. For example, if I retire at age 70, I might only feel that I need my money to last for 25 years at most. So, I can choose the one- or two-asset portfolio and percentage withdrawal amounts I feel comfortable with that should last for 25 years. A 55-year old retiree would likely have a different perspective.

There are several possible future avenues of research in this field. Extensions of this paper can go in at least three directions.

1. Increase the potential portfolio choices to include three-, four-, or five-asset portfolios. This will greatly increase the complexity and potential choices for investors. For example, the increase from two assets to three increases the number of potential portfolios from 95 to 1,080. For those investors who like more data and more choices, this may be a fruitful avenue of research.

2. Include international assets, corporate bonds, and/or high-yield bonds as potential choices for the portfolio. International assets have characteristics that often are not well correlated to domestic assets and could make for a richer dataset and range of choices. Including international assets may be a complex task as well. I suggest including the MSCI All-World Index as far back as one can reasonably include it. However, international equities may be further parsed into regions and countries if desired as well which would increase the options significantly. Including corporate bonds and/or high yield bonds would allow for assets that have intermediate amounts of risk relative to the large cap stocks and the T-bonds. Presumably, these assets would allow for higher risk-return choices in some scenarios.

   Including any of these assets would likely reduce portfolio volatility risk, which may be useful to many retirees. A significant limiting factor when considering including more assets is the availability of data, especially real data that is not artificially backfilled.

3. Look at variable withdrawal scenarios, including increasing withdrawal rates and declining withdrawal rates throughout retirement. A more realistic approach to the problem of needing changing amounts of money at different stages of retirement is to allow for changing withdrawal amounts during retirement.

   One method that may work is a declining amount. For example, one might assume that the retiree needs 8% initially, but that amount decreases over time as the retiree becomes less active and adjusts to a withdrawal rate of 4% after 10 years in retirement.

   Another method that may work is to assume increasing withdrawal rates if the retiree believes that he or she will need more funds to cover expected healthcare costs later in

life. There are innumerable ways to make these types of assumptions, and although they may be more realistic, it is ultimately an exercise in customized assumptions.

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